GR — Exercise sheet 1

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Units

Exercise 1.1: *Geometric units*
GR equations are often expressed in geometric units $c = G = 1$. In order to convert "back" to nongeometrized units, one must identify the physical dimension of each quantity entering the equation and rescale by the appropriate conversion factors.

- What is the conversion factor for a quantity with dimension of time?
- What is the conversion factor for a quantity with dimension of mass?
- What is the conversion factor for a quantity with dimension of energy?
- What is the conversion factor for a quantity with dimension of angular momentum?
- What is the conversion factor for a quantity with dimension $[q] = L^s T^q M^q$?

Special Relativity

Exercise 2.1: *Relativistic Doppler factor*
In an internal frame S, a train of light waves of wavelength λ travels in the negative $x-$direction towards an observer at the coordinate origin. The loci of the wavecrests then satisfy an equation of the form $x = -ct + n\lambda$, $n \in \mathbb{Z}$. Sketch some of such loci on a Minkowski diagram. Show that an observer boosted along the $x-$axis with speed v measures the wavelength

$$\lambda' = \lambda\sqrt{\frac{c-v}{c+v}}.$$

Exercise 2.2: *Special relativity quiz*
Three events A,B,C, are seen by an observer O to occur in order ABC. Another observer O' sees them in order CBA. Is it possible that a third observer O'' sees the events in order ACB? Draw spacetime diagrams to support your conclusions.

Exercise 2.3: *Lorentz transformations*
Verify that

- the transformation

$$ct' = ct\cosh\phi - x\sinh\phi \quad (1)$$
$$x' = -ct\sinh\phi + x\cosh\phi \quad (2)$$

  preserves $s^2 = -c^2t^2 + x^2$.

- $\tanh\phi = v/c \Rightarrow$

$$\cosh\phi = \frac{1}{\sqrt{1-(v/c)^2}} =: \gamma \quad (3)$$
$$\sinh\phi = \frac{v/c}{\sqrt{1-(v/c)^2}} \; . \quad (4)$$

- $c \to \infty \Rightarrow v\gamma/c^2 \to 0$

- A 1D rod has length L_0 meassured in its rest frame S (*proper length*). Use Lorentz transformations to compute the rod's length L' measured in an inertial frame S' moving at speed v with respect to S in the direction defined by the extension of the rod.