GR — Exercise sheet 2

Néstor Ortiz
[nestor.ortiz@uni-jena.de, Abbeanum, office 202]
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Special Relativity

Exercise 1.1: *Hyperbolic orthogonality*
Two four-vectors a^μ, b^ν are orthogonal if $a_\mu b^\mu = 0$. Show that (a) the sum of two vectors can be spacelike, null or timelike independently of the nature of the vectors, and (b) non-spacelike vectors which are orthogonal to a given nonzero null vector, must be multiples of the null vector. (c) Find four linearly independent null vectors in the Minkowski space.

Exercise 1.2: *Muon lifetime*
Particle physicist are so used to setting c=1 that they measure mass in units of energy. In particular, they tend to use electron volts ($1\text{eV} = 1.6\times10^{-19}\text{J}$), or, more commonly, keV, MeV, and GeV (10^3eV, 10^6eV, and 10^9eV, respectively). The muon has been measured to have a rest mass of 0.106GeV, and a rest frame lifetime of 2.19×10^{-6} seconds. Imagine that such a muon is moving in a circular storage ring of a particle accelerator, 1km in diameter, such that the muon's total energy is 1000GeV. How long would it appear to live from the experimenter's point of view? How many radians would it travel around the ring?

Exercise 1.3: *Homogeneous Maxwell equation*
Show that the subset of Maxwell equations in vacuum

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B}, \tag{1}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \tag{2}$$

are equivalent to the homogeneous equation

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0, \tag{3}$$

where $F_{\mu\nu}$ are the components of the Maxwell/Faraday tensor. Show that Eq. (3) can be written more elegantly as $\partial_{[\lambda} F_{\mu\nu]} = 0$. Further, show that $F_{\mu\nu}$ can be written as

$\partial_\mu A_\nu - \partial_\nu A_\mu$, where A^ν denotes the components of the electromagnetic vector potential. Verify that, in the potential formulation of electrodynamics (i.e. in terms of A^ν), the homogeneous Maxwell equation (3) is trivially satisfied.

Exercise 1.4: *Lorentz force in SR*
Show that the spatial part of the vector

$$\mathcal{F}^\nu := q u_\mu F^{\mu\nu},$$

where u^μ is the four-velocity of a charged point particle with charge q, corresponds to the classical Lorentz force. Compute also the component $\mathcal{F}^0$ and give a physical interpretation for it.