GR — Exercise sheet 1

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Units

Exercise 1.1: Geometric units

GR equations are often expressed in geometric units $c = G = 1$. In order to convert "back" to nongeometrized (SI) units, one must identify the physical dimension of each quantity entering the equation and rescale by the appropriate conversion factors.

- What is the conversion factor for a quantity with dimension of time?
- What is the conversion factor for a quantity with dimension of mass?
- What is the conversion factor for a quantity with dimension of energy?
- What is the conversion factor for a quantity with dimension of angular momentum?
- What is the conversion factor for a quantity with dimension $[q] = L^p T^r M^s$?

Special Relativity

Exercise 2.1: Diagram of 1+1 Mikowski spacetime

Draw a spacetime diagram in 1+1D (1 time and 1 spatial dimension) in units such that $c = 1$. Then draw

(1) a light ray.

(2) three events separated from the event at the origin by a null, timelike, and spacelike interval respectively.

(3) the worldline of an object at rest.

(4) the worldline of an object that travels with constant velocity $v < 1$.

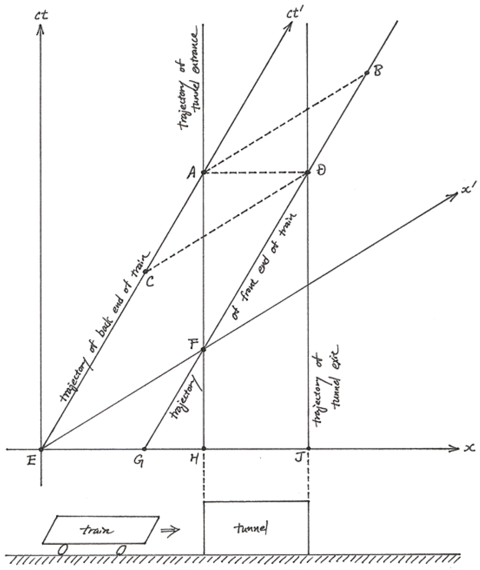

Figure 1: Spacetime diagram for the relativistic train problem.

(5) the worldline of an object that travels with constant velocity $v > 1$.

(6) the worldline of an accelerated object.

Briefly justify each draw.

Exercise 2.2: Relativistic train & the tunnel

A relativistic train speeds through a tunnel at constant velocity. The trajectories of both ends of the train and both ends of the tunnel are shown in the spacetime diagram.

(1) At which of the points labeled A through J in the space-time diagram does the nose of the train emerge from the tunnel?

(2) At which point does the tail of the train enter the tunnel?

(3) At which point is the tail of the train when the nose emerges from the tunnel in the train frame?

(4) At which point is the nose of the train when the tail of the train enters the tunnel in the tunnel frame?

(5) Does the train fit inside the tunnel in the tunnel frame? How about in the train frame? Do the two frames agree on this point? If not, explain why it is not a contradiction.

Exercise 2.3: Lorentz transformations

- Verify that the transformation

$$ct' = ct\cosh\phi - x\sinh\phi \tag{1}$$

$$x' = -ct\sinh\phi + x\cosh\phi \tag{2}$$

  preserves $s^2 = -c^2t^2 + x^2$.

- Define the velocity $\tanh\phi = v/c$, and verify that

$$\cosh\phi = \frac{1}{\sqrt{1-(v/c)^2}} =: \gamma \tag{3}$$

$$\sinh\phi = \frac{v/c}{\sqrt{1-(v/c)^2}} \; . \tag{4}$$

- $c \to \infty \Rightarrow v\gamma/c^2 \to 0$

- A 1D rod has length L_0 measured in its rest frame S (*proper length*). Use Lorentz transformations to compute the rod's length L' measured in an inertial frame S' moving at speed v with respect to S in the direction defined by the extension of the rod.