GR — Exercise sheet 2

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Special Relativity

Exercise 1.1: Relativistic Doppler factor

In an internal frame S, a train of light waves of wavelength λ travels in the negative $x-$direction towards an observer at the coordinate origin. The loci of the wavecrests then satisfy an equation of the form $x = -ct + n\lambda$, $n \in \mathbb{Z}$. Sketch some of such loci on a Minkowski diagram. Show that an observer boosted along the $x-$axis with speed v measures the wavelength

$$\lambda' = \lambda\sqrt{\frac{c-v}{c+v}}.$$

Exercise 1.2: Muon lifetime

Particle physicist are so used to setting c=1 that they measure mass in units of energy. In particular, they tend to use electron volts ($1\text{eV} = 1.6\times10^{-19}\text{J}$), or, more commonly, keV, MeV, and GeV (10^3eV, 10^6eV, and 10^9eV, respectively). The muon has been measured to have a rest mass of 0.106GeV, and a rest frame lifetime of 2.19×10^{-6} seconds. Imagine that such a muon is moving in a circular storage ring of a particle accelerator, 1km in diameter, such that the muon's total energy is 1000GeV. How long would it appear to live from the experimenter's point of view? How many radians would it travel around the ring?

Exercise 1.3: Homogeneous Maxwell equation

Show that the subset of Maxwell equations in vacuum

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B}, \quad (1)$$
$$\vec{\nabla} \cdot \vec{B} = 0, \quad (2)$$

are equivalent to the homogeneous equation

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0, \quad (3)$$

where $F_{\mu\nu}$ are the components of the Maxwell/Faraday tensor. Show that Eq. (3) can be written more elegantly as $\partial_{[\lambda}F_{\mu\nu]} = 0$. Further, show that $F_{\mu\nu}$ can be written as $\partial_\mu A_\nu - \partial_\nu A_\mu$, where A^ν denotes the components of the electromagnetic vector potential. Verify that, in the potential formulation of electrodynamics (i.e. in terms of A^ν), the homogeneous Maxwell equation (3) is trivially satisfied.

Exercise 1.4: Lorentz force in SR

Show that the spatial part of the vector

$$\mathcal{F}^\nu := q u_\mu F^{\mu\nu},$$

where u^μ is the four-velocity of a charged point particle with charge q, corresponds to the classical Lorentz force. Compute also the component $\mathcal{F}^0$ and give a physical interpretation for it.

Bonus exercise (2 points)

Exercise 2.1: Special relativity quiz

Three events A,B,C, are seen by an observer O to occur in order ABC. Another observer O' sees them in order CBA. Is it possible that a third observer O'' sees the events in order ACB? Draw spacetime diagrams to support your conclusions.