GR — Exercise sheet 6

Sarp Akcay
[sarp.akcay@uni-jena.de, Abbeanum, office 202]
(Return date: 02.12.19)

22.11.2019

Christoffel symbols

Exercise 1.1: Christoffel symbols in two dimensions

Compute $\Gamma^{\rho}_{\mu\nu}$ for the metric $g = dr^2 + r^2 d\theta^2$. [Hint: consider the nonzero components of metric and inverse metric, e.g., $g_{rr} = 1$, simplify the equations, count the nonzero symbols]

Geodesics

Exercise 2.1: Geodesic equations of motion from the Lagrangian approach

Consider the Lagrangian for a particle on generic spacetime:

$$L = \sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}} \ .$$

Show that the timelike geodesic equation can be derived from the variation of L. Note that the variation of the Lagrangian is a useful and alternative method to compute the Christtofel symbols for a given metric: starting from the Lagrangian one can compute the equations of motion and read-off the symbols fromt he result.

Exercise 2.2: Geodesic equations of motion using the Lagrangian approach

Let's do Robertson-Friedmann-Walker cosmology as a precursor to later chapters. Consider the following 1+1 dimensional metric

$$g = -dt^2 + a(t)^2(1 - kr^2)^{-1}dr^2$$

(a) Using the Lagrangian approach, derive the geodesic equations for the t and r coordinates from Lagrange's equations.

(b) Check the geodesic equations by computing the Christoffel coefficients ${\Gamma^{\alpha}}_{\beta\gamma}$ for this spacetime and inserting them into the standard geodesic equation.

Riemann tensor

Exercise 3.1: Riemann tensor

Compute ${R_{abc}}^d$, R_{ab} and R for (M, g) with

- $g = dx^2 + dy^2$
- $g = a^2(d\theta^2 + \sin^2\theta d\phi^2)$ with $a \in \mathbb{R}^+$ (2-sphere of radius a)
- $g = r^2 d\phi^2 + dz^2$ (Cylinder of constant radius r)

Note: though you may find some of the results in this exercise trivially disappointing. Think of these as practice rounds for similar calculations in more interesting spacetimes.