These semi-private notes are constructed from the following books:

- E.Di Casola et al Note on equivalence principles [https://arxiv.org/abs/1310.7426]
- C.Will The Confrontation between General Relativity and Experiment Living Review Relativity

If you decide to use them to study or teach, please
(0) be careful and refer to the original books
(1) cite/refer to my website
(2) let me know and send feedbacks.

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EQUIVALENCE PRINCIPLES

Roles of equivalence principles :

- Foundation of the theory
- Euristic principles (based on experimental facts)
- "Inspirational" principles

... according to the different views. In all the cases they :

- Help understanding theory and equations
- Provide us with the basis for <u>experimental tests</u> of the theory
- Provide us with a common ground to compare and connect different theories and formulations.

WEAK EQUIVALENCE PRINCIPLE (WEP)

Test-bodies with negligible self-gravity behave, in a gravitational field, independently of their properties.
("Universality of free fall")

Def : test-body = does not back-react on the gravitational field

Def : Measure of self-gravity $\sigma \equiv \frac{GM}{c^2 r}$, where M is the body mass and r its size.

- Negligible self-gravity means $\sigma \ll 1$.
- Test-body essentially depend on the external grav. field

} 2 independent concepts!

Examples

- Pebble in $\oplus$ grav. field : test-body and $\sigma \ll 1$

- $☾$ in $\oplus$ grav. field : $\sigma_{☾} \ll 1$ but cannot be considered a test body since it affect the field on Earth (e.g. tides)

- Micro black-hole $m_{BH} \sim m_{Planck} \approx 22 \mu g$ in $\oplus$ grav. field : $\sigma_{BH} \approx \frac{1}{2}$ self-gravity is large! But can be considered a test body on Earth.

NEWTON EQUIVALENCE PRINCIPLE (NEP)

In the Newtonian limit, the inertial and gravitational mass are equal.

Observations

- m_g and m_i are quantities defined in the Newtonian limit only
- Any theory of gravity must have the same Newtonian limit
- WEP $\Rightarrow$ NEP ...
- ... but in general NEP $\not\Rightarrow$ WEP because WEP validity depends on the equations of motion!

For Newton law :

$$m_i \ddot{\vec{x}} = - m_g \frac{GM\vec{x}}{x^3}$$

$$\ddot{\vec{x}} = - \frac{m_g}{m_i} \frac{GM\vec{x}}{x^3} \quad , \quad \text{NEP} \Rightarrow \text{WEP} \quad ✓$$

For any law that depends on $\frac{m_g}{m_i}$, e.g. $\ddot{\vec{x}} = -(1+\alpha)\frac{GM\vec{x}}{x^3}$, NEP $\Rightarrow$ WEP ✓

But if other combinations/terms are allowed might not be true.

Experimental tests of WEP

$$\alpha \equiv \frac{m_g}{m_i} \qquad a_A : \text{acceleration of body } A \quad (A = 1, 2)$$

$$\Delta a = |a_1 - a_2| \propto |\alpha_1 - \alpha_2|$$

<u>Eötvos parameter</u> : $\eta \equiv 2 \dfrac{|a_1 - a_2|}{|a_1 + a_2|} = 2 \dfrac{|\alpha_1 - \alpha_2|}{|\alpha_1 + \alpha_2|} = \underbrace{f\left(\dfrac{m_g}{m_i}\right)}$

fractional acceleration between two bodies

"Null" experiments to verify: $f(1) = 0$

Experiments :

- Newton pendulum $\quad \eta \sim 10^{-3}$
- Eötvos torsion balance $\quad \eta \sim 10^{-9}$
- MICROSCOPE (2018) $\quad \eta \sim 10^{-15}$

Note: Different formalisms have been developed to study parametric deviations from equivalence principles. Many of those accomodate the predictions of different theories of gravity for the same parameters.

For a review see : C. Will, Living Review Relativity.

EINSTEIN EQUIVALENCE PRINCIPLE (EEP)

Fundamental non-gravitational test particles and fields are, <u>locally</u> and at any point of spacetime, not affected by the presence of a gravitational field.

Main idea: local frames in grav. field $\Longleftrightarrow$ local frames in absence of gravity.

<u>Examples</u>

- locally non-rotating free-falling frame in grav. field = local inertial frame in absence of gravity
- local frame in grav. field = (suitably chosen) "accelerated" frame in absence of gravity that simulate "g"

- Meaning of LOCAL:

i) Sufficiently small region of spacetime such that a given instrument does not resolve variations of g (grav. field) and/or tidal forces.

Note however that the relative, fractional acceleration between two free-falling particles,

$$\frac{\ddot{S}}{S} \sim -\frac{\partial^2 \phi}{\partial x^i \partial x^j}$$

is governed by the tidal <u>tensor</u>, which in general does <u>not</u> vanish even for (an inhomogeneous ϕ) $S \to 0$.

Geodesic deviation and "composed system" in general violate EEP, strictly speaking.

ii) The "locality" requirement should be then interpreted together with the request of <u>fundamental</u> particle and fields.

Note that also with ii) there are difficulties: we do not know in general what "is fundamental" law or field. We know, however, and in many cases what is NOT fundamental; and we can discuss case-by-case ...

- Formulation of EEP for Poincaré invariant physics

The EEP formulated above is general.
We know however that "fundamental" non-gravitational physics laws are invariant under translations and the action of Lorentz group.
The EEP can be thus formulated more specifically as follows:

1. WEP is valid
2. LOCAL LORENTZ INVARIANCE (LLI)
3. LOCAL POSITION INVARIANCE (LPI)

LLI : Local non-gravitational experiments are independent of the velocity of the freely-falling frame in which they are performed.

LPI : Local non-gravitational experiments are independent of where and when in the Universe are performed.

Experimental tests of LLI

- Michelson-Morley
- High-energy astrophysical photons (GRB, ~ GeV) [arxiv: 0908.1832] FERMI

Note these are essentially tests of SR and/or quantum gravity.
In particular, quantum gravity theories assert that there exist a fundamental lenght scale:

$$\ell_{Planck} \equiv \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33} \text{ cm}$$

<u>Example</u> : Lorentz violating dispersion relations

According to SR : $E^2 = p^2 c^2$ $\rightarrow$ $v_\gamma = \frac{\partial E}{\partial p} = c$, light speed.

One could <u>postulate</u> phenomenological dispersion relations :

$$E^2 = p^2 c^2 (1 + \text{corrections})$$

$$= p^2 c^2 + E_{Planck} f_1 |p| c + f_2 p^2 c^2 + f_3 E_{Planck}^{-1} |p|^3 c^3 + \dots$$

and constrain the corrections with experiments :

$$\frac{v_\gamma}{c} = 1 + \text{... corrections ...}$$

f_i : parameters

$$E_{Planck} = \sqrt{\frac{\hbar c^5}{G}} \sim 10^{19} \text{ GeV}$$

For example γ-photons of different energies would arrive at different times.
Effect is maximize for $E \approx E_{Planck}$ ...

Experimental tests of LPI

- Gravitational redshift, Pound - Rebka.

  $z = \frac{\Delta\phi}{c^2} \longrightarrow z = (1+\alpha)\frac{\Delta\phi}{c^2}$, constrain the deviation α.

- Shift of spectral lines due to the $\odot$ grav. field.
  (complex due to Doppler shift related to turbulence / convection in the photosphere)

- clocks on satellite

- GPS, relativistic effects $\approx 36\,\mu s = \underbrace{49\,\mu s}_{\text{grav. redshift GR}} - \underbrace{7\,\mu s}_{\text{time dilation SR}}$

STRONG EQUIVALENCE PRINCIPLE (SEP)

The EEP does not include gravitational phenomena. An extension of EEP is SEP:

> All fundamental <u>test</u> physics is not affected locally by the presence of a gravitational field.

SEP includes <u>test</u> but <u>self-gravitating</u> bodies ($\sigma \sim 1$).
An alternative formulation is:

1. WEP is valid for self-gravitating and test bodies
2. Any local test experiment is independent of the velocity of the free-falling apparatus
3. Any local test experiment is independent of when and where in the Universe it is performed.

<u>Examples</u> of test gravitational experiments

- Cavendish experiments; mutual attraction between two light bodies
- Gravitational wave detection
- Any experiment in which a <u>background</u> gravitational field can be identified and the latter does not affect the gravitational measure.

Experimental tests of SEP

- Violation of WEP for gravitating bodies that induce orbit perturbations
- Location and frame-dependent effects in the measurement of G

Example: Nordvedt effect

The acceleration of a body of grav. mass m_g and inertial mass m_i in an external grav. field ϕ is:

$$\ddot{\vec{x}} = \frac{m_g}{m_i} \nabla\phi$$

and one could parametrize:

$$\frac{m_g}{m_i} = 1 - \eta_N \frac{E_g}{m} \qquad E_g > 0$$

where $E_g = -$ gravitational self-energy of the body.

For laboratory-size bodies: $\frac{E_g}{m_i} \leq 10^{-27}$ => no effect in Eötvös experiments.

For astronomical bodies: $\frac{E_g}{m_i} \sim 10^{-6}$ $\odot$ Sun

$\sim 10^{-8}$ ♃ Jupiter

$\sim 10^{-10}$ $\oplus$, ☾ Earth, Moon,

so $\eta_N \neq 0$ might give a significant / measurable effect.

For example, if $\eta_N \neq 0$ Earth would "fall" toward the Sun with a slightly different acceleration than the Moon.
Lunar laser ranging experiments, started with the Apollo 11 mission, indicate that $\eta_N < 10^{-12}$.

CONSEQUENCES OF EQUIVALENCE PRINCIPLES

Let us consider the consequences of postulating EP for the development of theories of gravity, by assuming only to describe "gravity as spacetime geometry":

- WEP $\Rightarrow$ Worldlines of test bodies depend only on the gravitational field, not on their properties.

  The gravitational field is thus associated with a set of curves whose tangent vector is autoparallel $\rightarrow$ geodesics, that identify test-bodies trajectories.

  If there is no gravitational field, than these geodesics are straight lines corresponding e.g. to Newtonian trajectories or worldlines in SR.

  If there is a gravitational field, than the geodesics are generically those of a dynamical and curved spacetime.

Note at this stage the "dynamical and curved" spacetime is unspecified.

- EEP (In absence of gravity the physical law are those in the framework of SR)

  $\Rightarrow$ The structure of spacetime must be locally Minkowskian.

$$g \sim \eta \qquad \text{and} \qquad \Gamma = 0$$

  This is possibile only if g is a Lorentzian metric and Γ (or ∇) is the associated Levi-Civita connection associated to g.

  Moreover,

  $\Rightarrow$ the solutions of the theory involving non-gravitational phenomena must be locally the same as those in SR.

  This point hints to how the dynamical laws should be written in the theory.

Summarizing, EEP $\Rightarrow$ gravity can be described by <u>metric theories</u>:

1. there exist a symmetric metric
2. Test bodies follow geodesics of the metric
3. In Local Lorentz frames, the non-gravitational laws of physics are those of SR.

<u>Observations</u>

- 2.,3. $\Rightarrow$ particles and non-gravitational fields all couple in the same way to a single gravitational field.
  This <u>Universal coupling</u> allows one to consider the metric has a property of the spacetime rather than field on it.

- In metric theories of gravity only the metric couple to the non-gravitational fields (or "the matter"). However, there could be other fields (scalar, vectors, ...) whose role could be to mediate the way matter generate the gravitational field and produce the metric.
  (Note once g is determined, g alone tells matter how to move ...)

  Metric theories of gravity differ in the way additional gravitational fields are introduced.
  Will divides them in two classes:

  i) PURELY DYNAMICAL
  Metric and additional gravity field are determined dynamically by field equations.

  ii) PRIOR GEOMETRIC
  Fields or other elements are given "a priori".
  <u>Examples</u>: - a background metric
  - absolute time coordinate

Finally, what is the role of SEP?

SEP $\Rightarrow$ There is only one gravitational field represented by the metric g.

A gravity theory satisfying SEP is a PURE METRIC theory.

Physically, any non-purely metric theory predicts that the mass-energy of self-gravitating bodies acquires a dependence on the extra gravitational fields. That produces a force which makes the motion non-geodesics in a non-trivial background.

Exercise: Gravitational redshift

Problem: A photon is emitted at p with frequency ν_p and received at q with frequency ν_q. Determine the relation between ν_p and ν_q observed by stationary observers with 4-velocity $u^a = (u^0, \vec{0})$.

Let us assume the gravitational field is described by the metric

$$g = -(1+2\phi)\,dt^2 + (1-2\phi)(dx^2 + dy^2 + dz^2)$$

where ϕ is time independent and $|\phi(x,y,z)| \ll 1$. We work at first order in ϕ.
$x^\mu = (t,x,y,z)$ is a global coordinate system reducing to inertial coords for $\phi = 0$.

The metric above is a <u>static</u> and <u>weak</u> gravitational field.

Photon trajectories are <u>null geodesics</u> in the metric g : $x^\mu(\lambda)$, where λ is the affine parameter. The tangent vector is the photon's 4-momentum : $p^\mu = \frac{dx^\mu}{d\lambda}$.
The energy of the photon measured by the observers with 4-velocity u^μ is:

$$E = p_\mu u^\mu .$$

Note that $u^\mu = (u^0, \vec{0})$ and $u_\mu u^\mu = -1 \Rightarrow g_{00} u^0 u^0 = -1 \Rightarrow u^0 = \frac{1}{\sqrt{-g_{00}}} = \frac{1}{\sqrt{1+2\phi}}$.
At first order in ϕ: $u^0 \simeq 1 - \phi$.
Hence the energy is:

$$E = p_\mu u^\mu = p_0 u^0 \simeq p_0 (1-\phi)$$

We need to calculate p_0. Consider the geodesic equation for p_μ:

$$\frac{dp_\mu}{d\lambda} = \frac{d}{d\lambda}\left(g_{\mu\sigma} p^\sigma\right) = \underbrace{\frac{dg_{\mu\sigma}}{d\lambda} p^\sigma}_{= \frac{\partial g_{\mu\sigma}}{\partial x^\nu} \frac{dx^\nu}{d\lambda} p^\sigma} + g_{\sigma\mu} \underset{\substack{\uparrow \\ \text{substitute} \\ \text{geodesic eq.}}}{\frac{dp^\sigma}{d\lambda}} = \cancel{\partial_\nu g_{\mu\sigma} p^\nu p^\sigma} + \frac{1}{2} g_{\alpha\beta,\mu} p^\alpha p^\beta - \cancel{\partial_\alpha g_{\beta\mu} p^\alpha p^\beta} \Rightarrow$$

$$\frac{dp_\mu}{d\lambda} = \frac{1}{2} g_{\alpha\beta,\mu} p^\alpha p^\beta$$

$\mu = 0$:
$$\frac{dp_0}{d\lambda} = \frac{1}{2} g_{\alpha\beta,0} p^\alpha p^\beta \underset{\uparrow}{=} 0 .$$

$g_{\alpha\beta}$ is independent on $x^0 = t$

Thus,

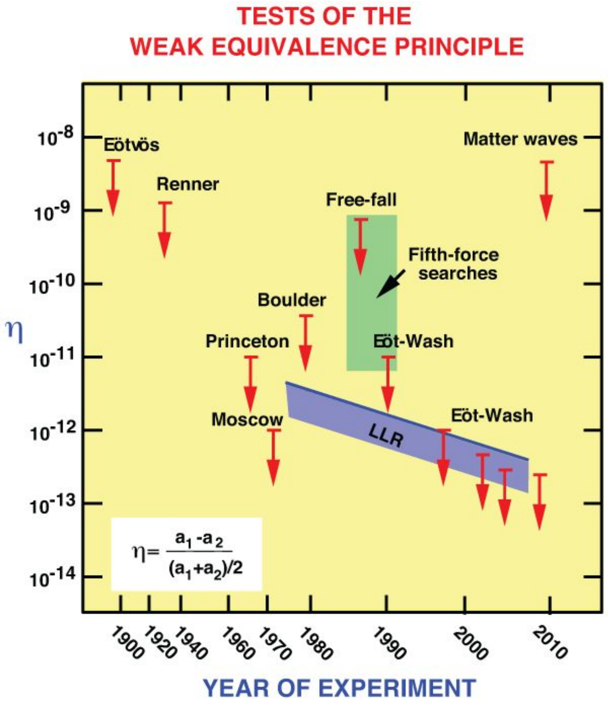
$$\frac{dp_0}{d\lambda} = 0 \quad \Rightarrow \quad p_0(p) = p_0(q) \equiv \bar{p}_0$$

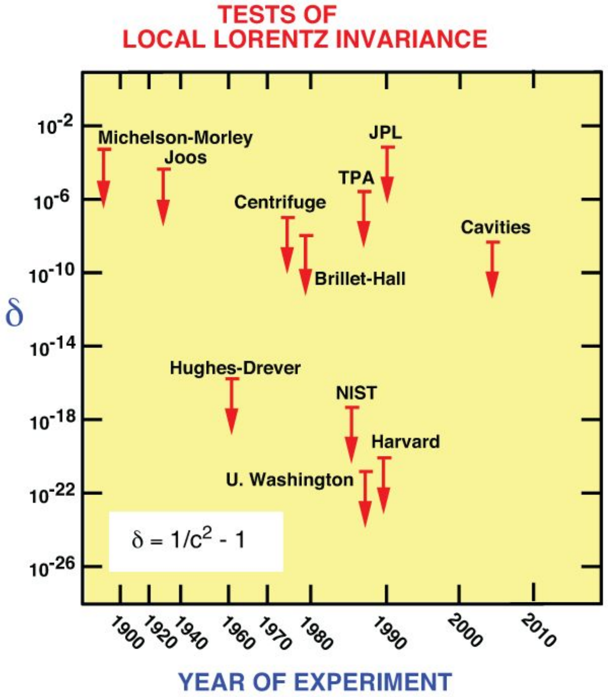
Let us take the ratio of frequencies / energies :

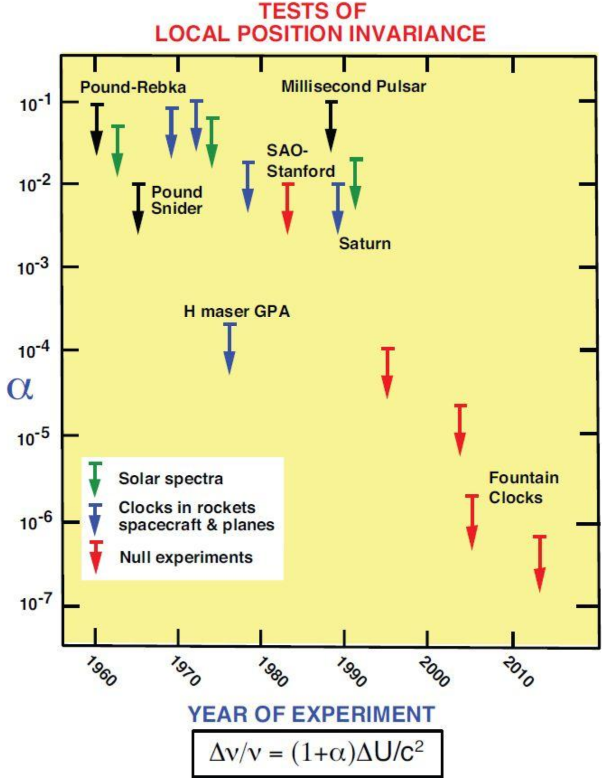
$$\frac{\nu_q}{\nu_p} = \frac{E(q)}{E(p)} = \frac{p_\mu u^\mu(q)}{p_\mu u^\mu(p)} = \frac{\cancel{\bar{p}_0}\, u^0(q)}{\cancel{\bar{p}_0}\, u^0(p)} \cong \frac{1-\phi(q)}{1-\phi(p)} \cong 1 - \Delta\phi$$

with $\Delta\phi = \phi(q) - \phi(p)$.

Tests of the Foundations of Gravitation Theory







https://link.springer.com/article/10.12942/lrr-2014-4