

Geometry of Robertson-Walker spacetime

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Collection of general formulas for Robertson-Walker spacetime in $n = 4$ dimensions. These calculations are preparatory to discuss RFW cosmology.

GR class @ Jena FSU, WS 2018/19 <https://bernuzzi.gitlab.io/gr/>

Follow notation of [1].

Assume the metric:

$$g = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \quad (1a)$$

with the 2-sphere metric

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \quad (1b)$$

Christoffel symbols (only nonvanishing components):

$$\Gamma_{rr}^t = \frac{a\dot{a}}{1 - kr^2} \quad \Gamma_{rr}^r = \frac{kr}{1 - kr^2} \quad \Gamma_{tr}^r = \dot{a}a^{-1} \quad (2a)$$

$$\Gamma_{t\phi}^\phi = \Gamma_{t\theta}^\theta = \Gamma_{tr}^r \quad \Gamma_{\theta\theta}^t = a\dot{a}r^2 \quad \Gamma_{\phi\phi}^t = \Gamma_{\theta\theta}^t \sin^2 \theta \quad (2b)$$

$$\Gamma_{r\theta}^\theta = \Gamma_{r\phi}^\phi = r^{-1} \quad \Gamma_{\theta\theta}^r = -r(1 - kr^2) \quad \Gamma_{\phi\phi}^r = \Gamma_{\theta\theta}^r \sin^2 \theta \quad (2c)$$

$$\Gamma_{\phi\phi}^\theta = -\sin \theta \cos \theta \quad \Gamma_{\phi\theta}^\phi = \cot \theta \quad (2d)$$

Ricci tensor:

$$R_{tt} = -3\ddot{a}a^{-1} \quad R_{rr} = \frac{a\ddot{a} + 2\dot{a} + 2k}{1 - kr} \quad (3a)$$

$$R_{\theta\theta} = r^2(a\ddot{a} + 2\dot{a}^2 + 2k) \quad R_{\phi\phi} = R_{\theta\theta} \sin^2 \theta \quad (3b)$$

Ricci scalar:

$$R = 6 \left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right] \quad (4a)$$

[1] S. M. Carroll, "Spacetime and geometry: An introduction to general relativity," San Francisco, USA: Addison-Wesley (2004) 513 p