Geometry of Robertson-Walker spacetime

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Collection of general formulas for Robertson-Walker spacetime in $n = 4$ dimensions. These calculations are preparatory to discuss RFW cosmology.
GR class @ Jena FSU, WS 2019/20 `http://sbernuzzi.gitpages.tpi.uni-jena.de/gr/`

Follow notation of [1].
Assume the metric:

$$g = -\mathrm{dt}^2 + \mathrm{a}^2(\mathrm{t}) \left(\frac{\mathrm{dr}^2}{1 - \mathrm{kr}^2} + \mathrm{r}^2 \mathrm{d}\Omega^2 \right) \tag{1a}$$

with the 2-sphere metric

$$\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2 \tag{1b}$$

Christoffel symbols (only nonvanishing components):

$$\Gamma^t_{rr} = \frac{a\dot{a}}{1 - kr^2} \qquad \Gamma^r_{rr} = \frac{kr}{1 - kr^2} \qquad \Gamma^r_{tr} = \dot{a}a^{-1} \tag{2a}$$

$$\Gamma^\phi_{t\phi} = \Gamma^\theta_{t\theta} = \Gamma^r_{tr} \qquad \Gamma^t_{\theta\theta} = a\dot{a}r^2 \qquad \Gamma^t_{\phi\phi} = \Gamma^t_{\theta\theta}\sin^2\theta \tag{2b}$$

$$\Gamma^\theta_{r\theta} = \Gamma^\phi_{r\phi} = r^{-1} \qquad \Gamma^r_{\theta\theta} = -r(1 - kr^2) \qquad \Gamma^r_{\phi\phi} = \Gamma^r_{\theta\theta}\sin^2\theta \tag{2c}$$

$$\Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta \qquad \Gamma^\phi_{\phi\theta} = \cot\theta \tag{2d}$$

Ricci tensor:

$$R_{tt} = -3\ddot{a}a^{-1} \qquad R_{rr} = \frac{a\ddot{a} + 2\dot{a}^2 + 2k}{1 - kr^2} \tag{3a}$$

$$R_{\theta\theta} = r^2(a\ddot{a} + 2\dot{a}^2 + 2k) \qquad R_{\phi\phi} = R_{\theta\theta}\sin^2\theta \tag{3b}$$

Ricci scalar:

$$R = 6\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right] \tag{4a}$$

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[1] S. M. Carroll, "Spacetime and geometry: An introduction to general relativity," San Francisco, USA: Addison-Wesley (2004) 513 p