Gravitational waves — Exercise sheet n.1

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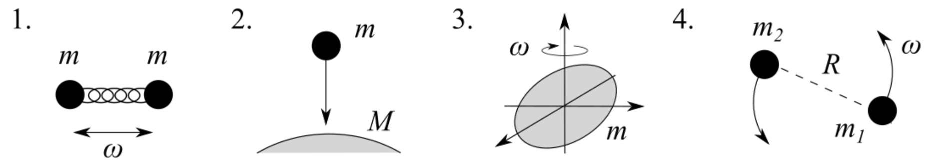
Exercise 1.1: Quadrupole approximation



Consider the following sources of gravitational radiation (see Fig. 1):

1. Two point particles with mass m oscillating with pulsation ω along a fixed axis;
2. Free-falling point-particle with mass m in a Newtonian gravitational field (of mass M);
3. Ellipsoid (with semi-axes a, b, c) rotating around one of its principal axis with frequency ω;
4. Two point particles (with different masses m_1, m_2) in Newtonian circular orbit.

For these cases, compute:

- Inertia tensor of the source,

$$Q^{ij}(t) \equiv I^{ij} - \frac{1}{3}\delta^{ij}\, I^{kk} = \int d^3x\, \rho(t,\vec{x}) \left(x^i x^j - \frac{1}{3} r^2 \delta^{ij} \right) . \qquad (1)$$

  Note that I_{ij} is the standard inertia tensor, while Q_{ij} is the trace-free inertia tensor.

- Gravitational wave emitted in quadrupole approximation in the TT gauge,

$$h_{ij}^{\rm TT}(t,\vec{x}) = \frac{2G}{r\, c^4}\, \Lambda_{ij,mn}(\theta,\phi)\, \ddot{Q}_{mn}(t - r/c)\, , \qquad (2)$$

  where $\Lambda_{ij,mn}$ is the TT projector.