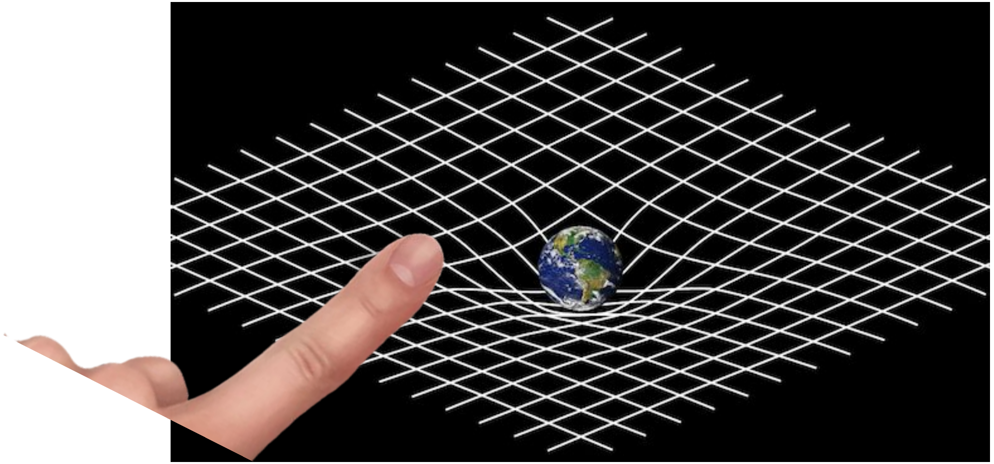
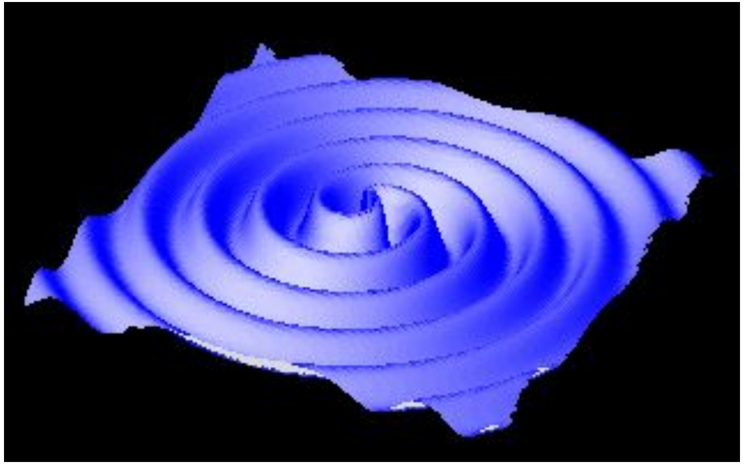
Gravitational waves
A crash course

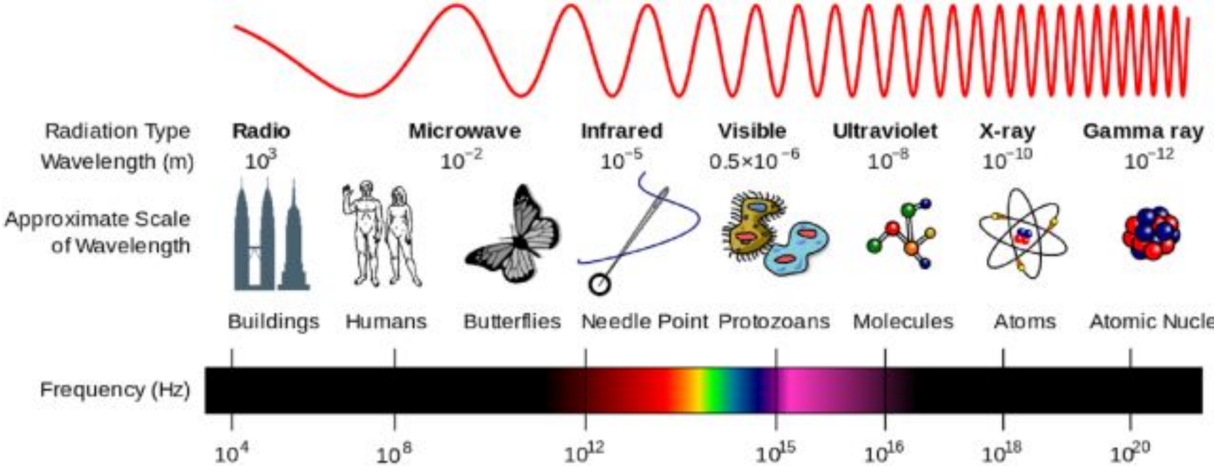
S.Bernuzzi

G. Carullo

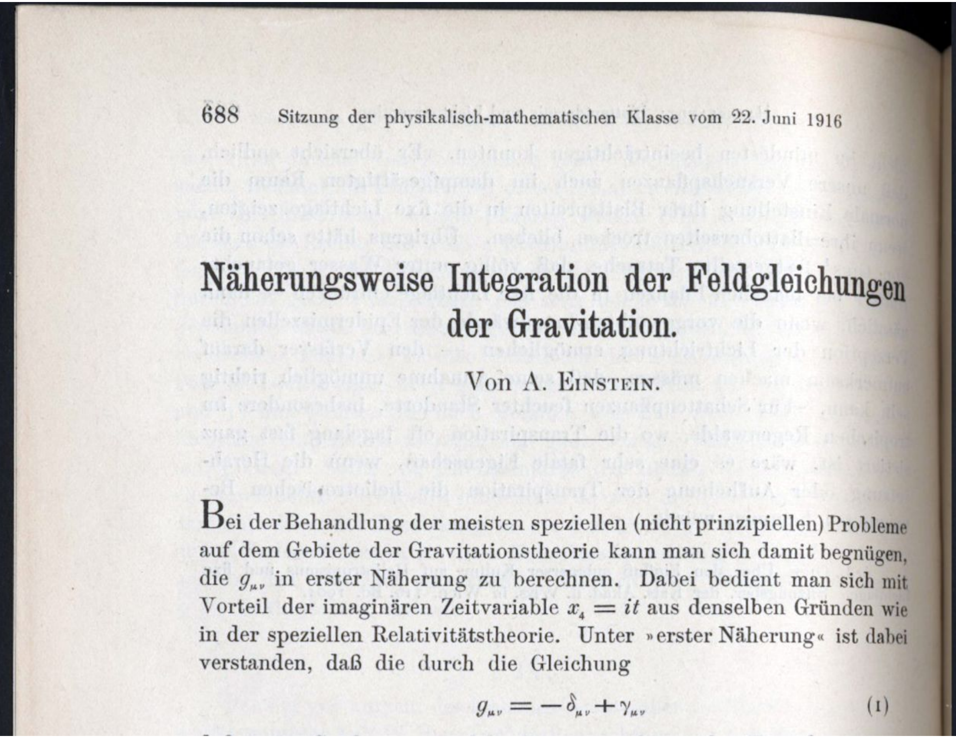
Ripples in the fabric of spacetime







Gravitational waves: A theoretical challenge



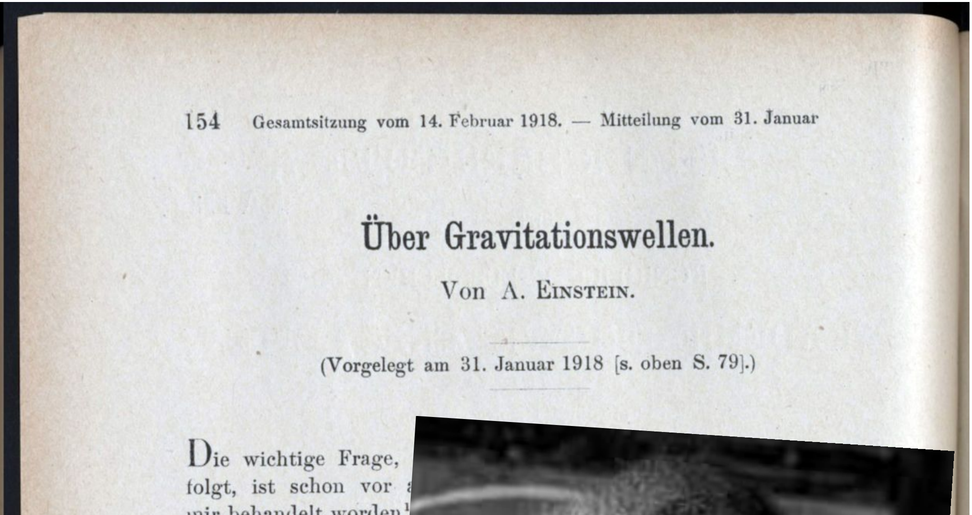
688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$



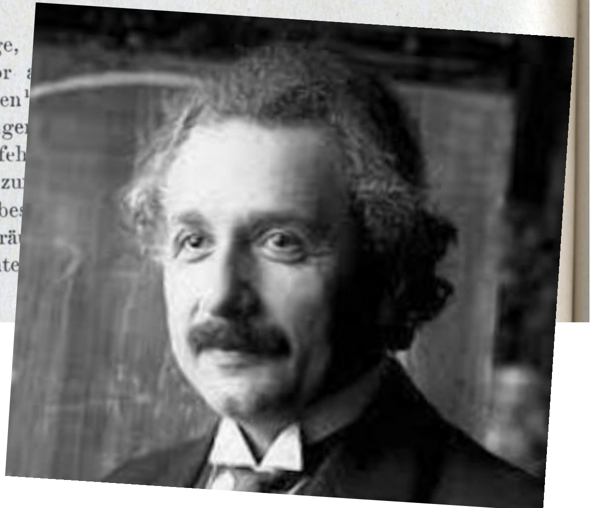
154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage,
folgt, ist schon vor
mir behandelt worden
standes nicht genüge
dauerlichen Rechenfeh
die Angelegenheit zu
Wie damals be
das betrachtete zeiträ
nur sehr wenig unte

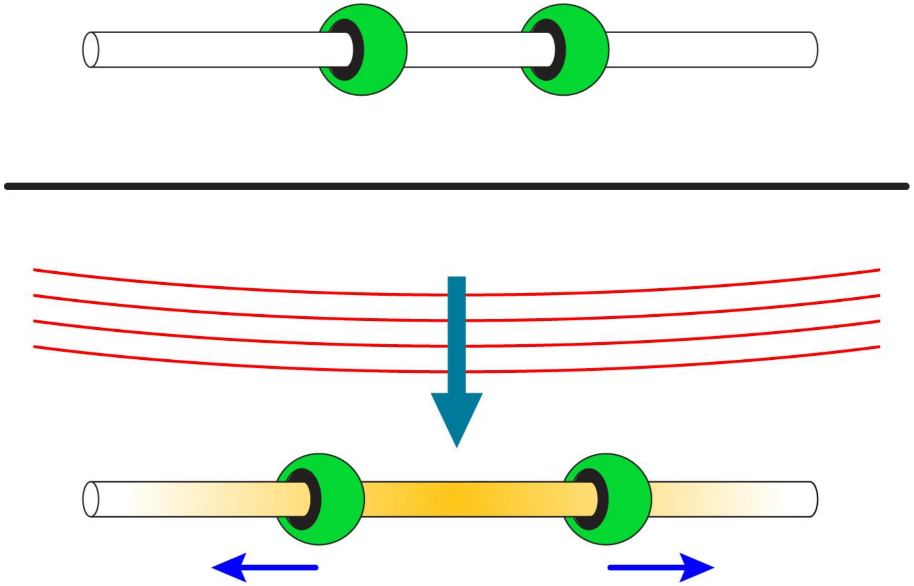


Gravitational waves: A theoretical challenge

Major contributions to radiation theory [from E.T.Newmann]

1. J. Goldberg, PR 99, 1873-83 (1955)
2. F. Pirani, (1956), Bull. Acad. Polo. Sci, III, 5, p. 143 (Introduced Petrov Classification of Algebra of the Weyl tensor.)
3. R. Sachs, P.G. Bergmann; 1958, Phys. Rev. 112, p. 674 (linear theory, definition of multipoles)
4. A. Trautman, King's College Notes; Lectures on General Relativity, 1958, eventually revised and published in "Lectures on General Relativity" Vol.1, Prentice Hall, 1965 (Sommerfeld radiation conditions applied to GR, very influential set of notes.) and recently republished as a "golden oldie" in the GRG Journal.
5. R. Penrose, Ann. Phys., 1960, 10, p. 171. (Major exposition of Spinor Calculus and GR)
6. R. Sachs, Proc. Roy. Soc., 1961, 264, p. 309 (Introduced optical parameters, shear, divergence, twist, asymptotic structure of curvature tensor)
7. F. Pirani, 1961, King's College Notes; published in "Lectures on General Relativity", Vol.1, Prentice Hall, 1965 (Introduce the Petrov Classification of Weyl tensors)
8. J. Goldberg, R. Sachs, (1962), Acta Physica Polonica, Vol. XII, p12 (The Goldberg–Sachs Theorem; Princ. Null Vectors of Algebraically Special Metrics)
9. H. Bondi, M. van der Burg, A. Metzner, 1962, Proc. Roy. Soc. 269, p.21, (Introduction of null coordinates, asymptotic solutions of Einstein equations, mass loss Theorem, BMS group)
10. R. Sachs, Proc. Roy. Soc., 1962, 270, p. 103 (Generalized Bondi work, elucidated the BMS group)
11. E. Newman, R. Penrose, JMP, 1962, 3, p.566 (systematic use of tetrad calculus and spinor analysis, Goldberg–Sachs Theorem, Peeling)
12. E. Newman and T. Unti, 1962, JMP, 3, p. 892 (Asymptotic Integration of the Einstein Eqs, the BMS group)
13. I. Robinson and A. Trautman, 1962, Proc. Roy. Soc. 265 p.463, (Integrated most of the Einstein Eqs. for twist-free algebraically special metrics)
14. R. Penrose, 1963, Phys.Rev. Lttrs, 10, p. 66, (Introduced Null Infinity and Conformal Compactification of Space-Time)

Sticky bead argument (Bondi/Feynmann - Chapel Hill)



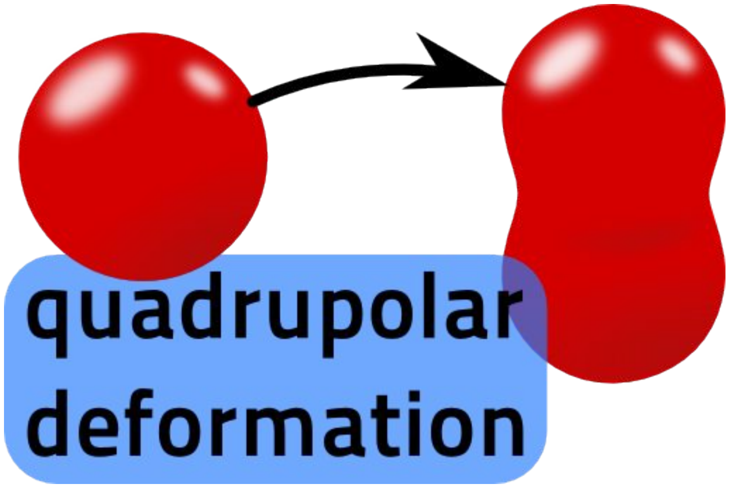
Gravitational waves: A theoretical challenge

PROPERTIES

- Transverse
- Speed of light
- Carry energy
- “plus” & “cross” polarization
- Quadrupole sources

Quadrupole formula :

$$\bar{h}_{ij}(t,r) = \frac{2G}{c^4 r}\ddot{I}_{ij}(t-r)$$



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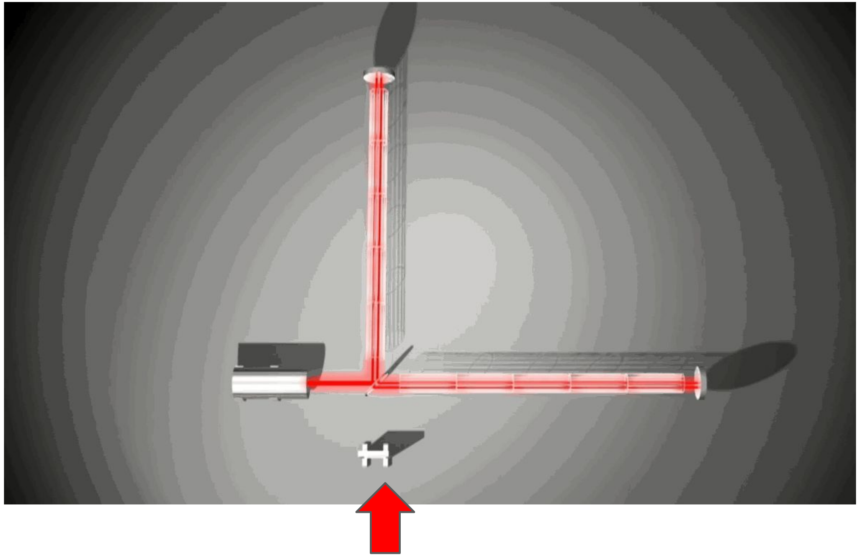
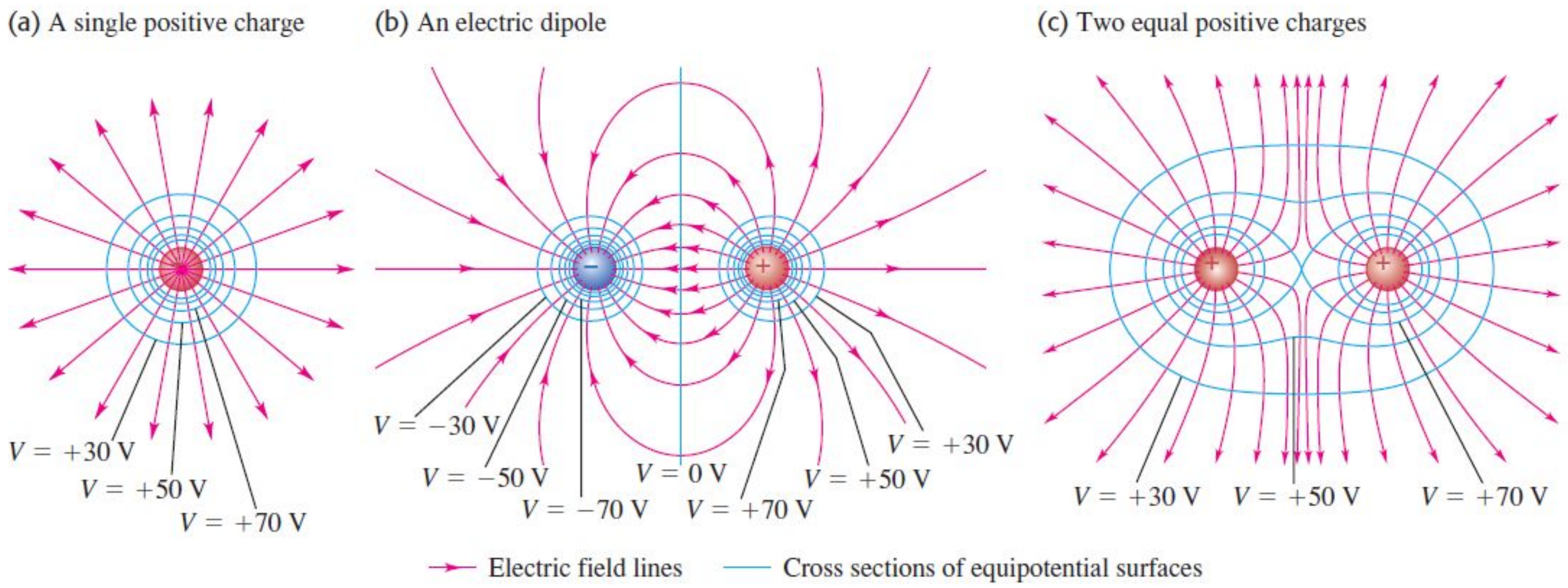
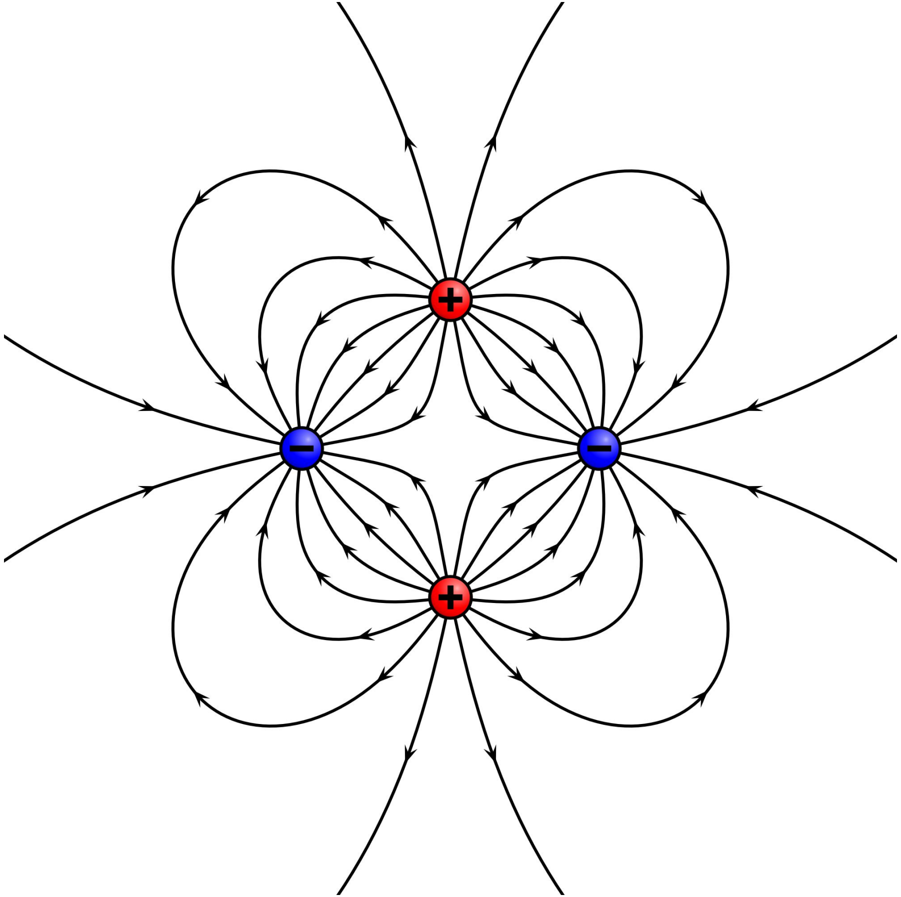


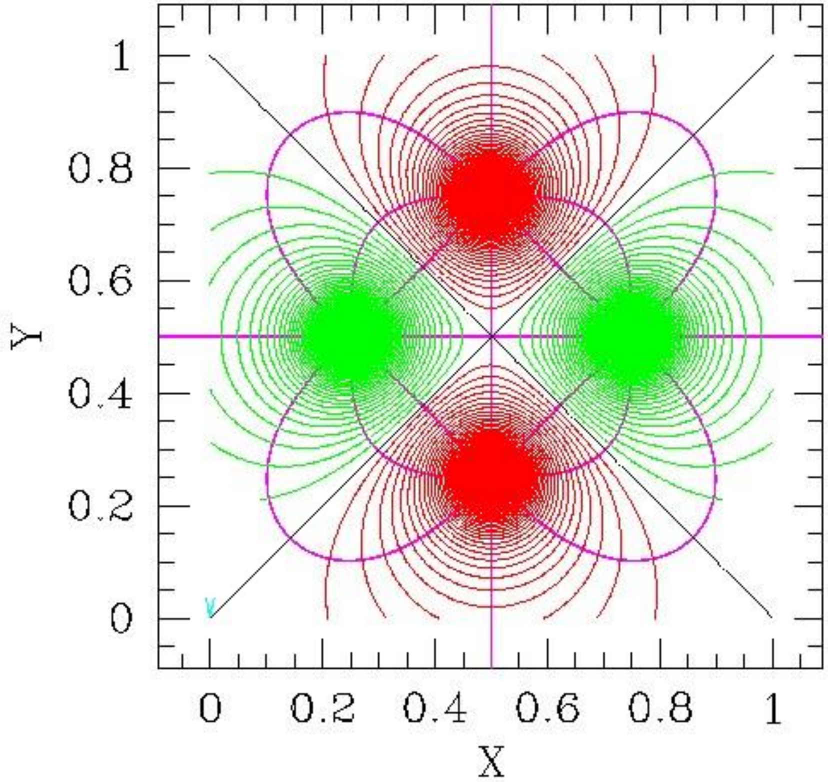
Potential of charge distributions, dipoles, ...

23.23 Cross sections of equipotential surfaces (blue lines) and electric field lines (red lines) for assemblies of point charges. There are equal potential differences between adjacent surfaces. Compare these diagrams to those in Fig. 21.28, which showed only the electric field lines.



… and the quadrupole!





Multipole expansion

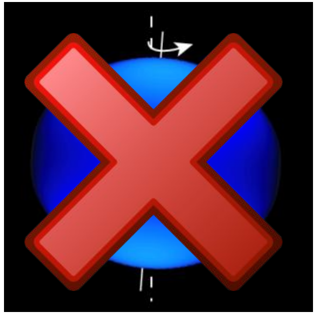
$$\Delta\phi = \rho \quad \Rightarrow \quad \phi(\vec{r}) = \int \frac{\rho(\vec{x})\ \mathrm{d}^3x}{|\vec{r} - \vec{x}|}$$

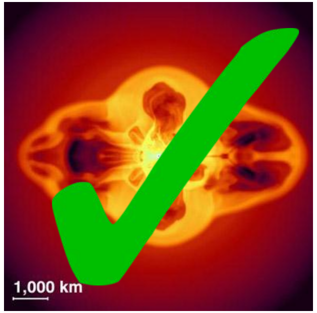
$$\phi(\vec{r}) = -\frac{q}{r} - \frac{p_i n^i}{r^2} - \frac{Q_{ij} n^i n^j}{r^3} + \ldots$$

Multiple moments of the mass/charge distribution

Intuition: Q_{ij} ~ deviation from spherical distribution along the axes

Quiz: Which one is a GW source?





Gravitational waves: An experimental challenge

$$h \lesssim \frac{GM}{c^2 D} \left(\frac{v}{c}\right)^2 = \left(\frac{R}{D}\right) \left(\frac{GM}{c^2 R}\right) \left(\frac{v}{c}\right)^2$$

GW strain

$$h \sim \frac{\delta L}{L}$$

- Car collision (D ~ m): h ~ 10^{-43}
- Supernova explosion (D ~ 10 kpc): h ~ 10^{-17}
- Black hole collision (D ~ 100 Mpc): h ~ 10^{-21}

Strongest sources:

- Big Bang
- Mergers of black holes and neutron stars)
- Supernovae

Gravitational waves: An experimental challenge

$$h \lesssim \frac{GM}{c^2 D}\left(\frac{v}{c}\right)^2 = \left(\frac{R}{D}\right)\left(\frac{GM}{c^2 R}\right)\left(\frac{v}{c}\right)^2$$

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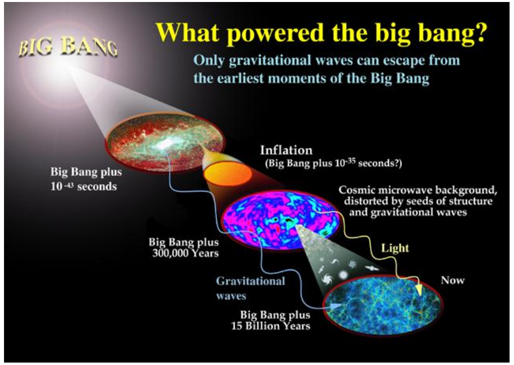
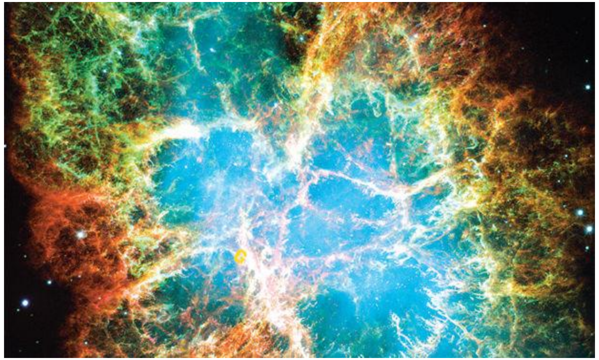
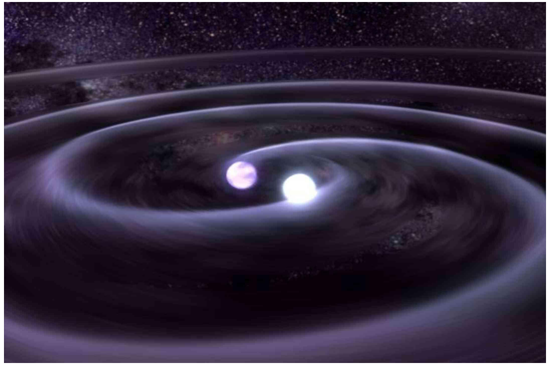
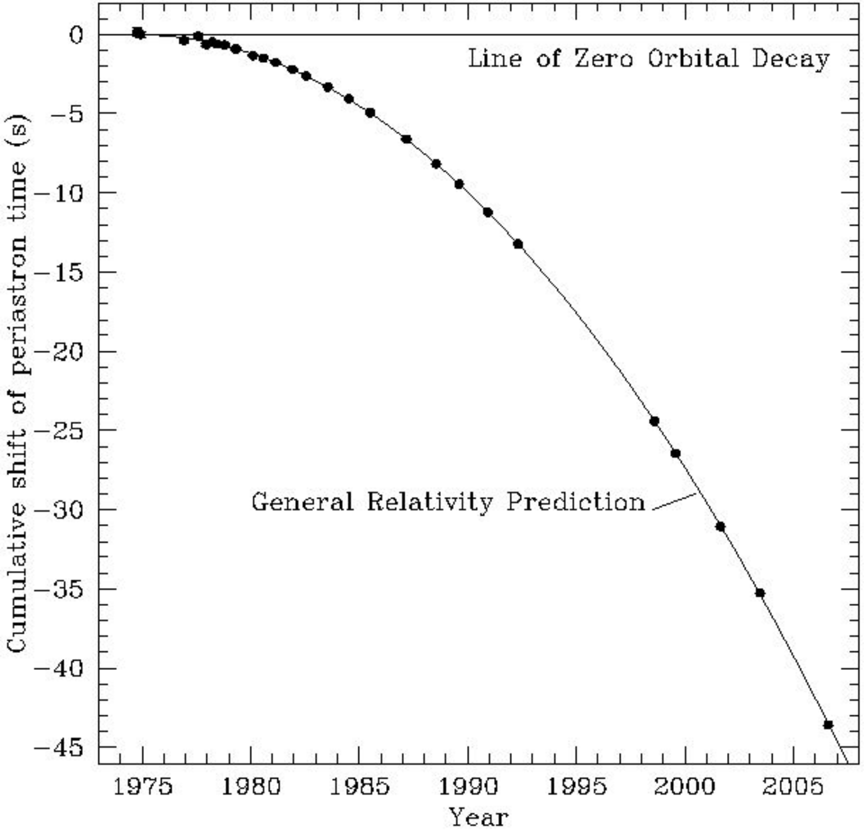
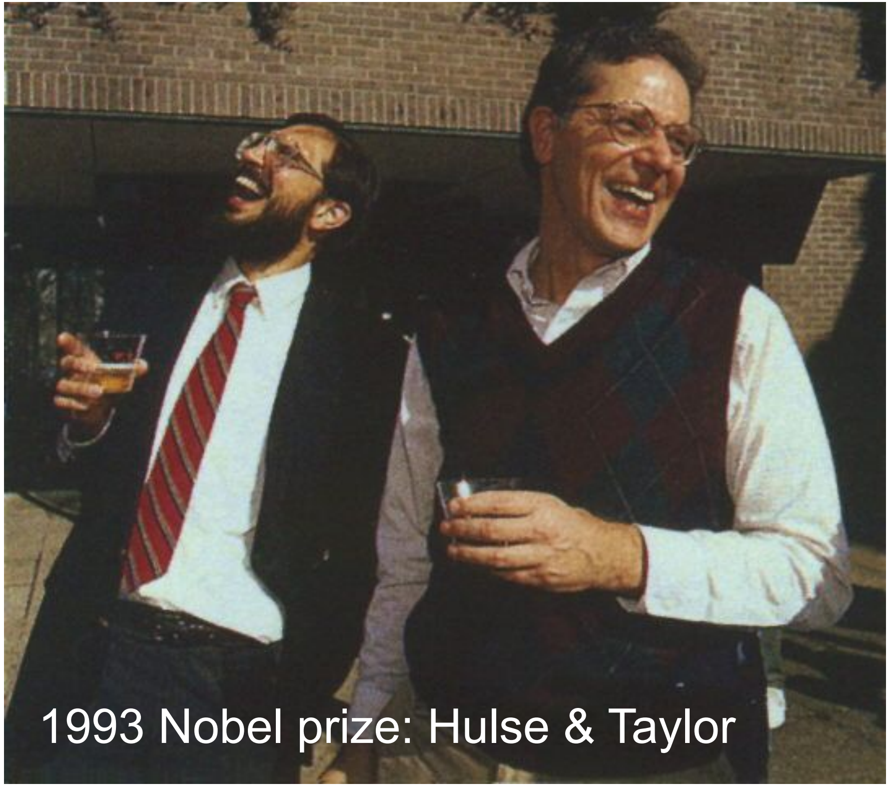






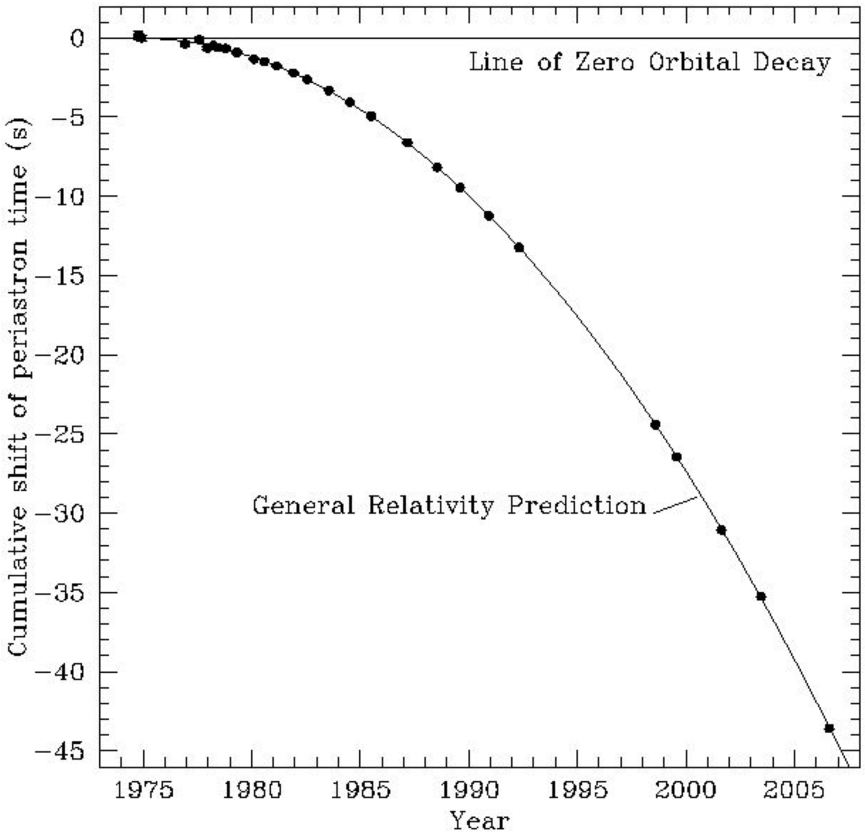
First observational evidence: PSR B1913+16

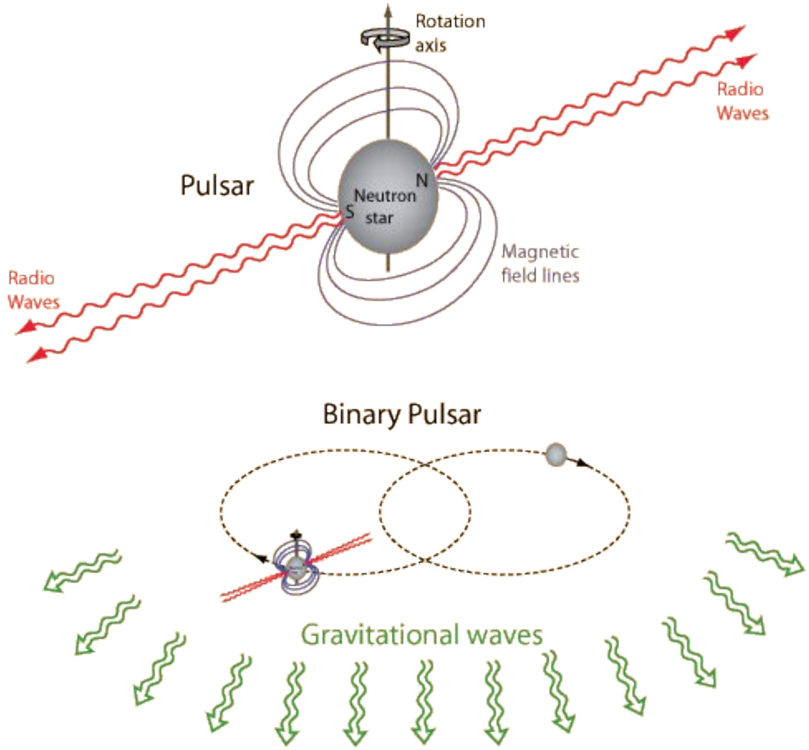




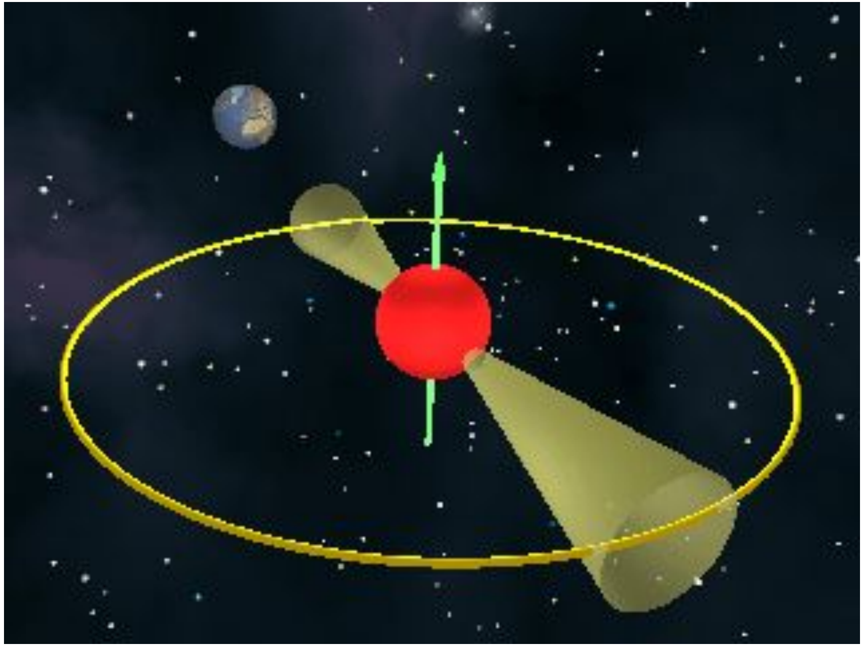
1993 Nobel prize: Hulse & Taylor

First observational evidence: PSR B1913+16

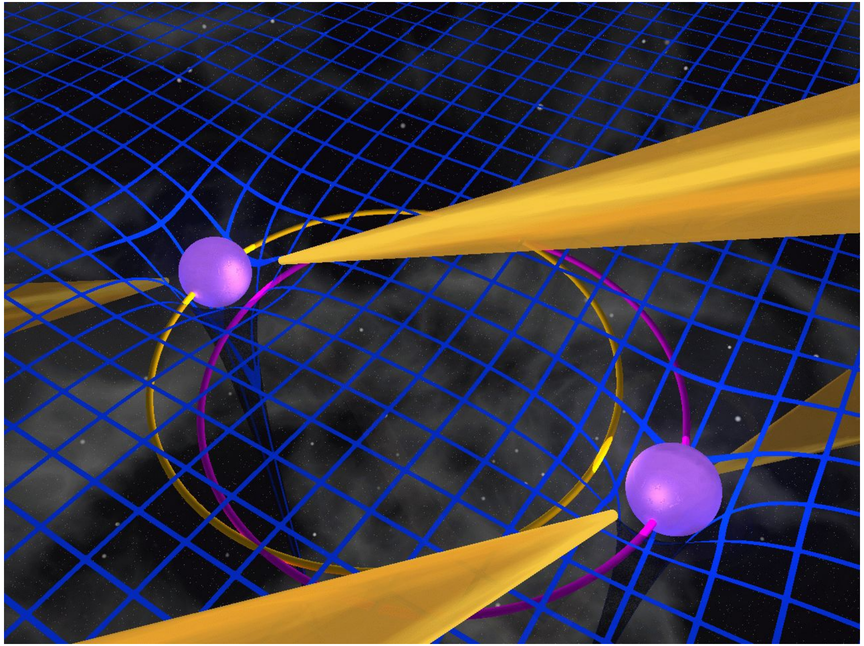




Neutron star, pulsars, binary systems

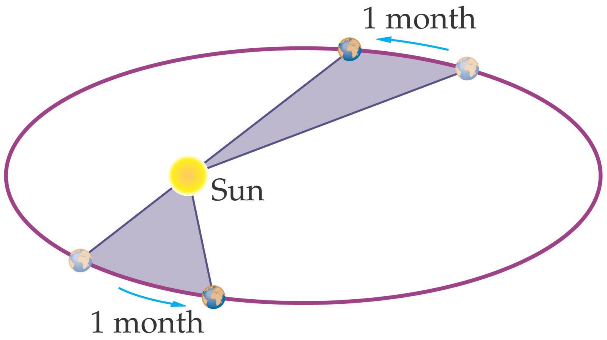


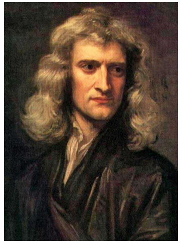
Single Pulsar



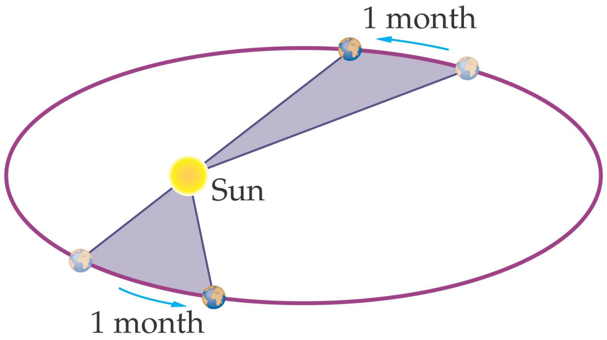
Double pulsar PSR **J0737−3039**

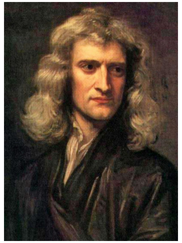
Two-body problem in general relativity

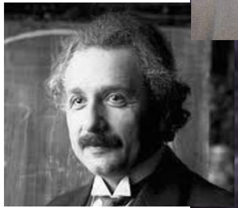


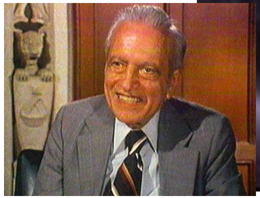


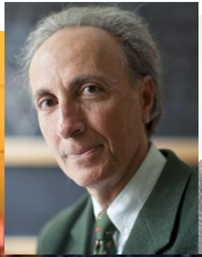
Two-body problem in general relativity

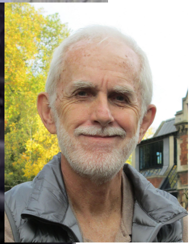












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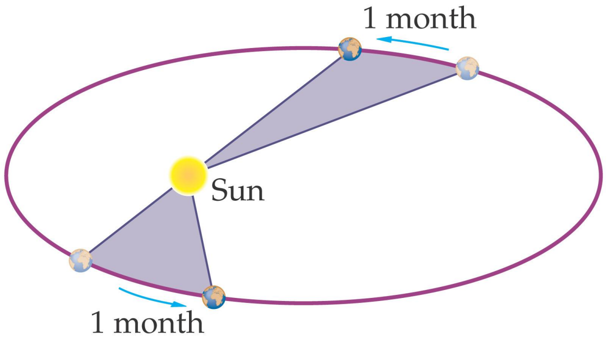
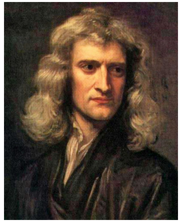
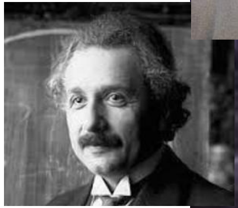
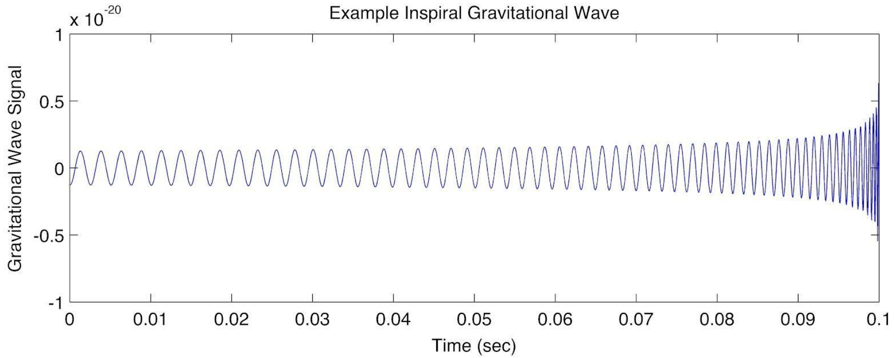
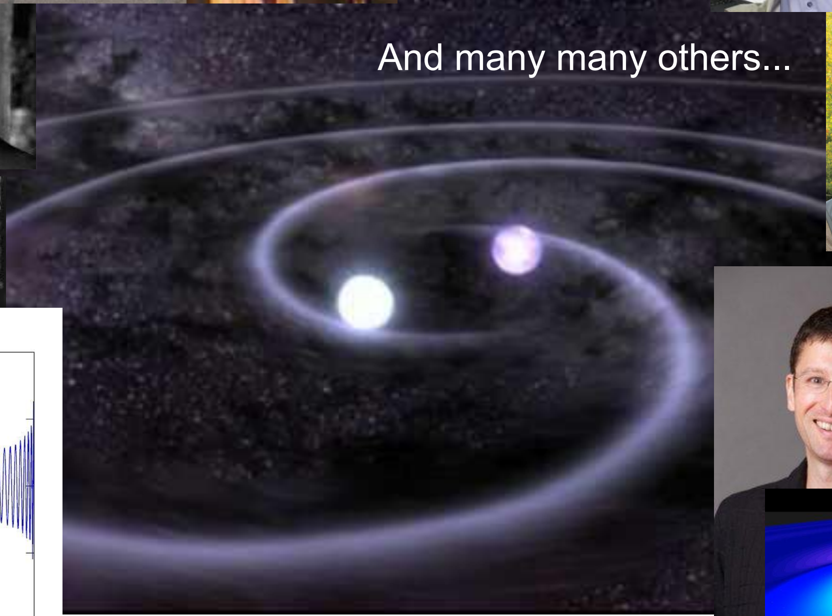
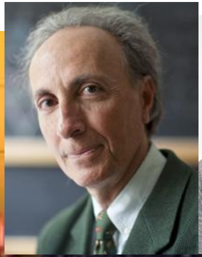


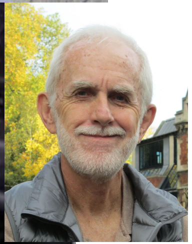


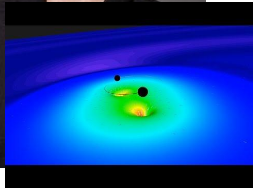






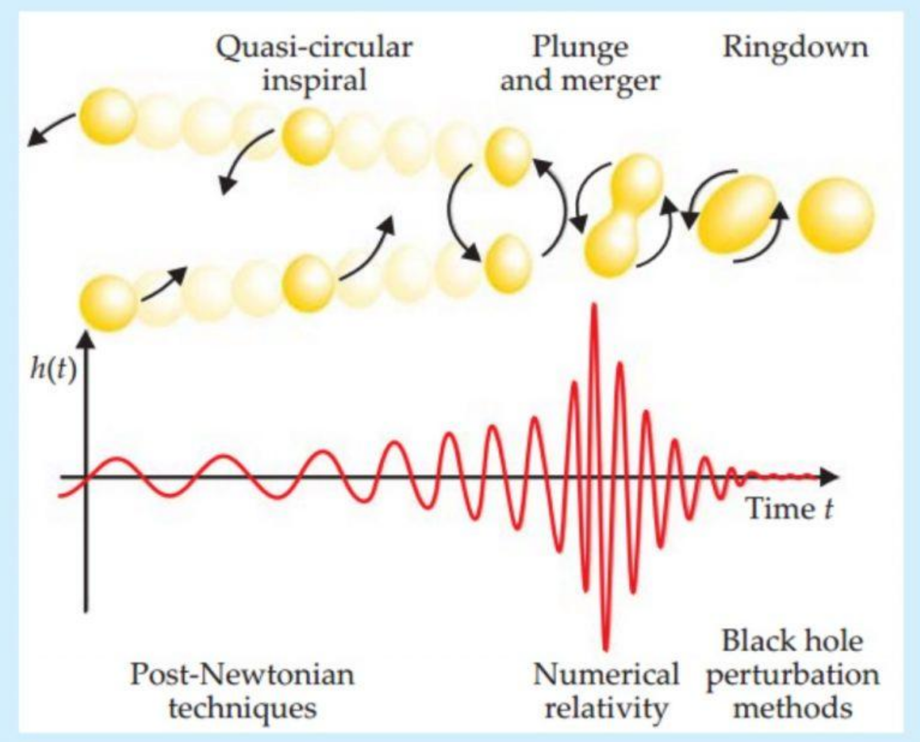






Waveform ID: Adiabatic inspiral

$$h_+(t) \simeq \frac{4}{r}\left[\frac{GM_c}{c^2}\right]^{5/3}\left[\frac{\pi f_{\rm gw}(t)}{c}\right]^{2/3}\underbrace{\cos\left[2\pi f_{\rm gw}(t)t\right]}_{:=\cos[\phi_{\rm gw}(t)]}$$



F.Dyson 1963 (Landau-Lifshitz 1951) → intense GW flash from coalescing binaries made of compact objects

Wave frequency is twice the orbit's frequency

Chirp mass:

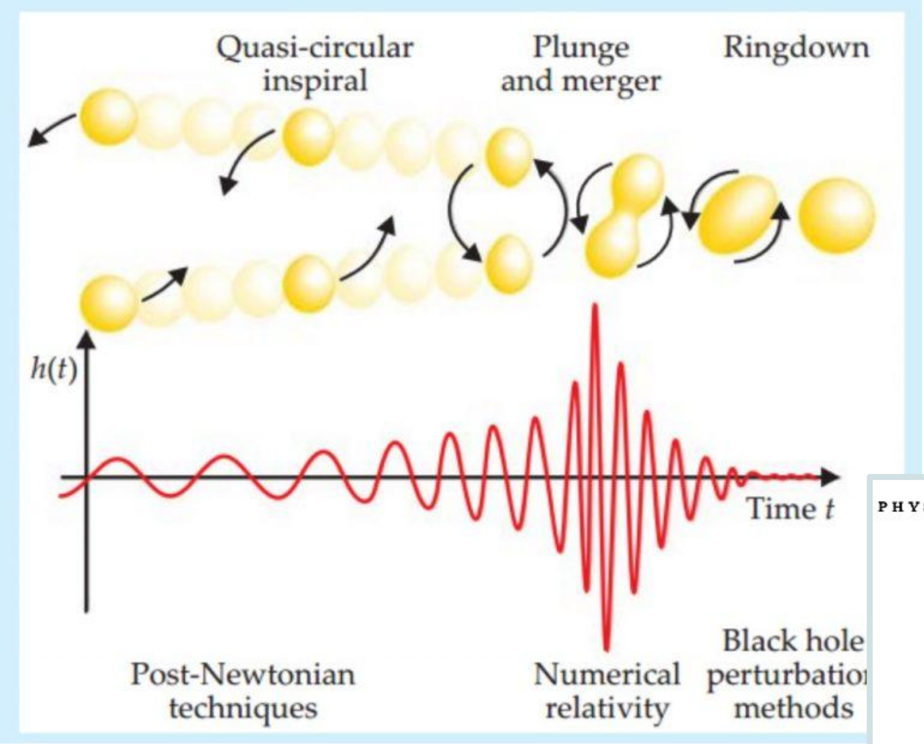
$$M_c := \mu^{3/5}M^{2/5} = \left(\frac{\mu}{M}\right)^{3/5} M = \nu^{3/5}M$$

Trivial mass scale:

$$h(t) \sim \frac{1}{r}M_c^{5/3}f_{\rm gw}^{2/3}(t) = \nu\frac{1}{r}M^{5/3}f_{\rm gw}^{2/3} = \nu\frac{1}{(r/M)}(Mf_{\rm gw}(t))^{2/3}$$

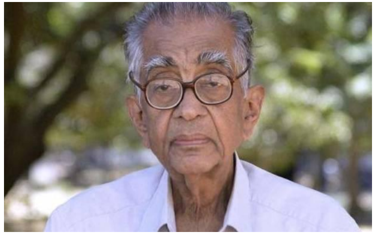
$$\phi_{\rm gw}(t) \sim 2\phi_{\rm orb}(t) = 2M_c^{-5/8}t^{5/8} = 2\nu^{-3/8}\left(\frac{t}{M}\right)^{5/8}$$

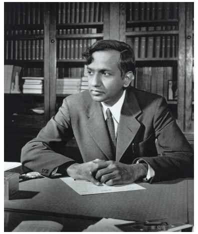
Waveform ID: The ringdown



"It was a natural question then to ask: how does one see a black hole? So, using a computer, I scattered packets of gravitational waves from a black hole and the quasinormal modes emerged carrying the signatures of the black hole… this was theoretical. I had never dreamed that this would take on an aspect of reality some day,"

Prof. Vishveshwara (6 March 1938 – 16 January 2017)
[www.thehindu.com]





PHYSICAL REVIEW — VOLUME 108, NUMBER 4 — NOVEMBER 15, 1957

Stability of a Schwarzschild Singularity

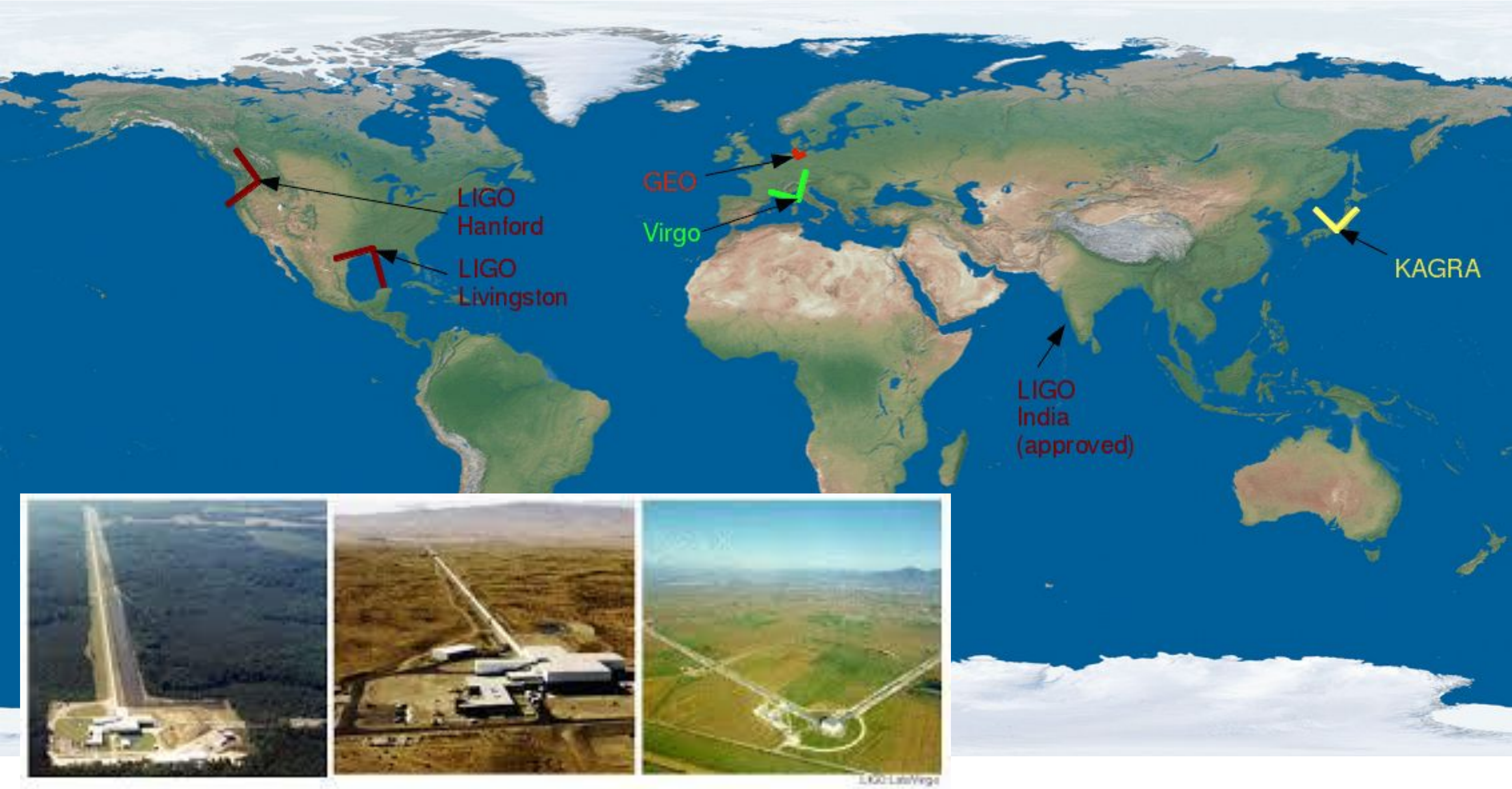
TULLIO REGGE, *Istituto di Fisica della Università di Torino, Torino, Italy*

AND

JOHN A. WHEELER, *Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*

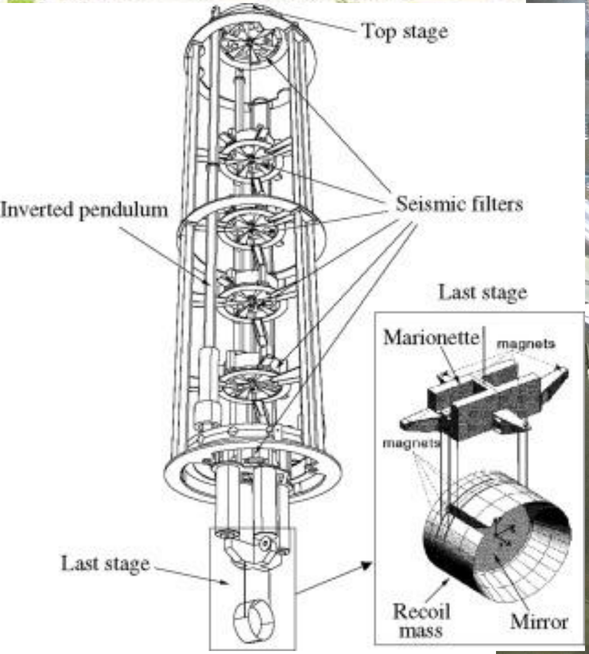
(Received July 15, 1957)

It is shown that a Schwarzschild singularity, spherically symmetrical and endowed with mass, will undergo small vibrations about the spherical form and will therefore remain stable if subjected to a small nonspherical perturbation.

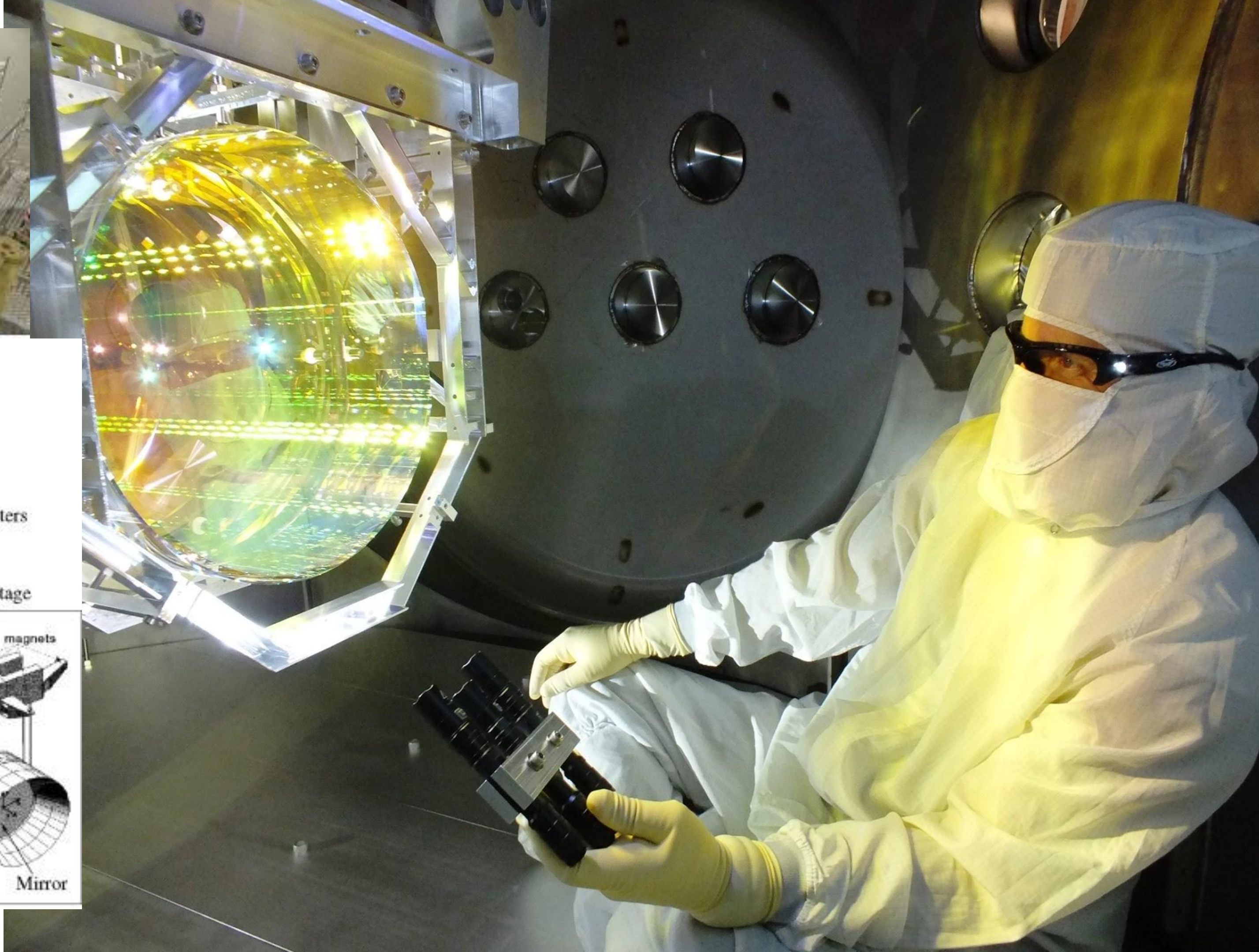

GEO
LIGO
Hanford
Virgo
LIGO
Livingston
KAGRA
LIGO
India
(approved)

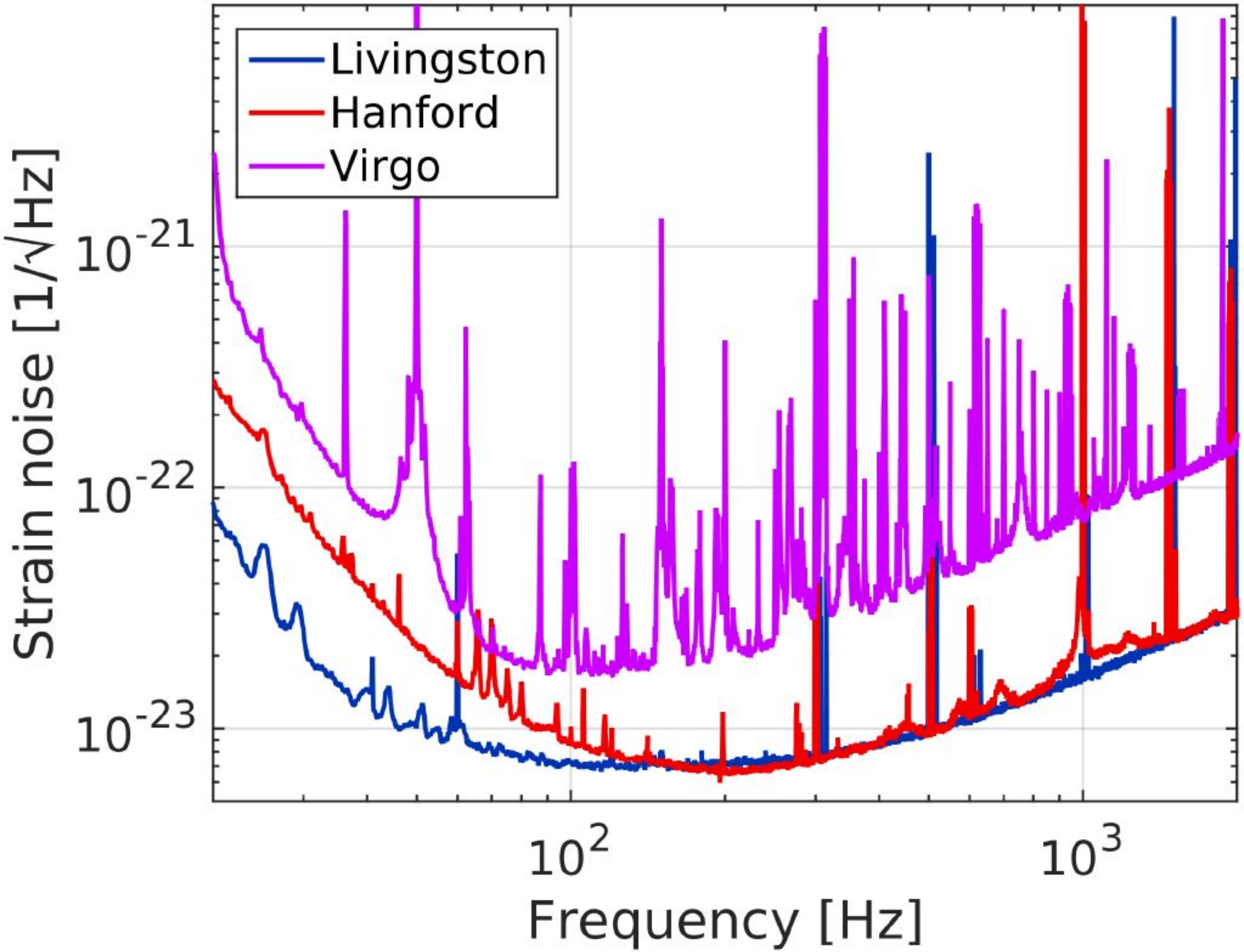
Virgo

European Gravitational Observatory (EGO),
Cascina, Pisa (Italy)



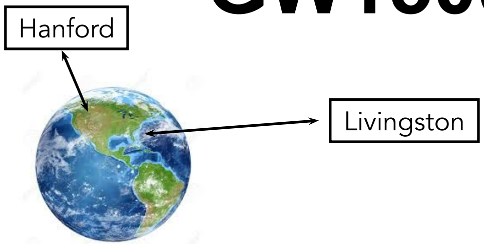
Top stage
Seismic filters
Inverted pendulum
Last stage
Last stage
Marionette
magnets
magnets
Recoil mass
Mirror

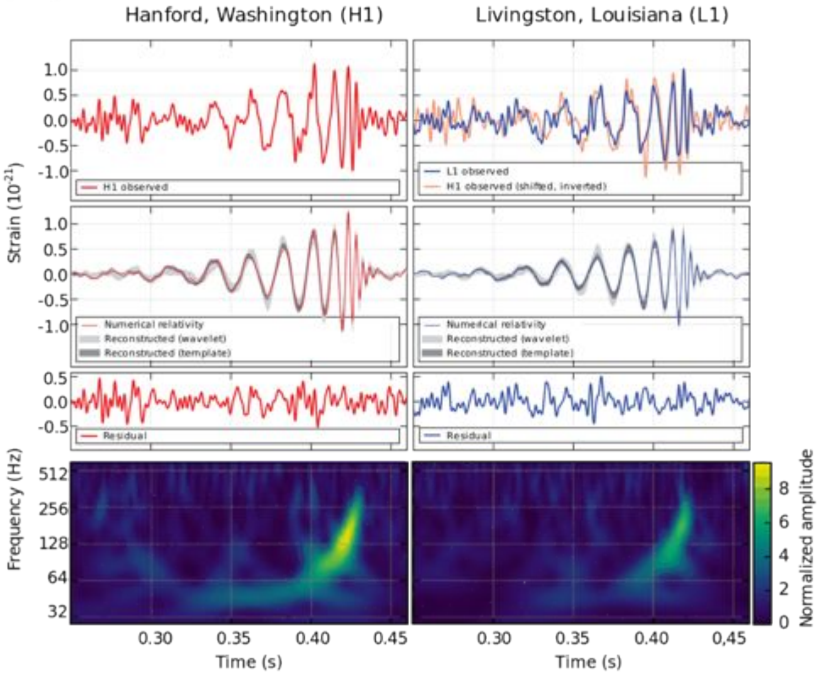


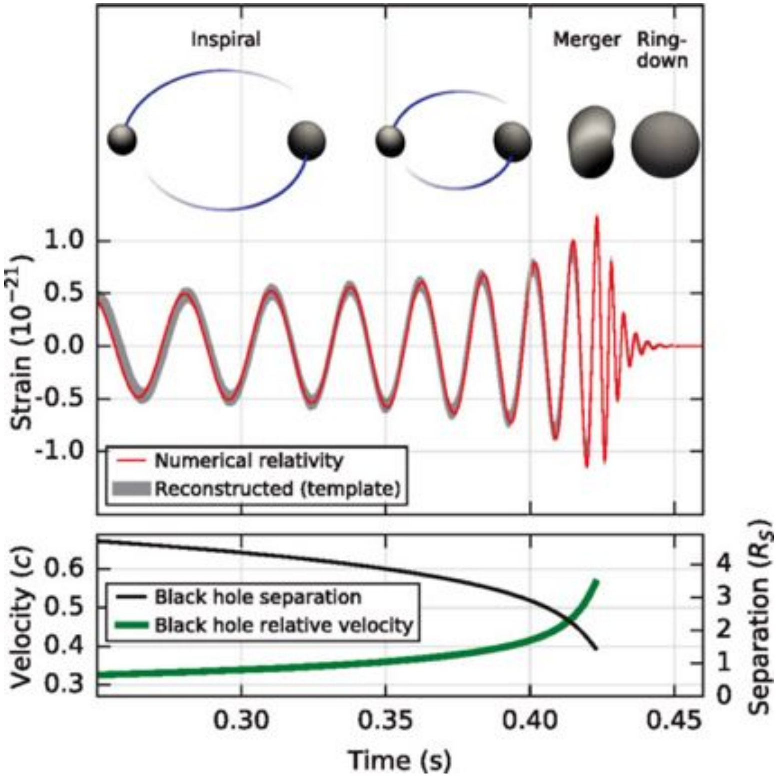

Livingston
Hanford
Virgo
Strain noise [1/√Hz]
10^-21
10^-22
10^-23
10^2
10^3
Frequency [Hz]

August 2017

GW150914: First wave detected on Earth

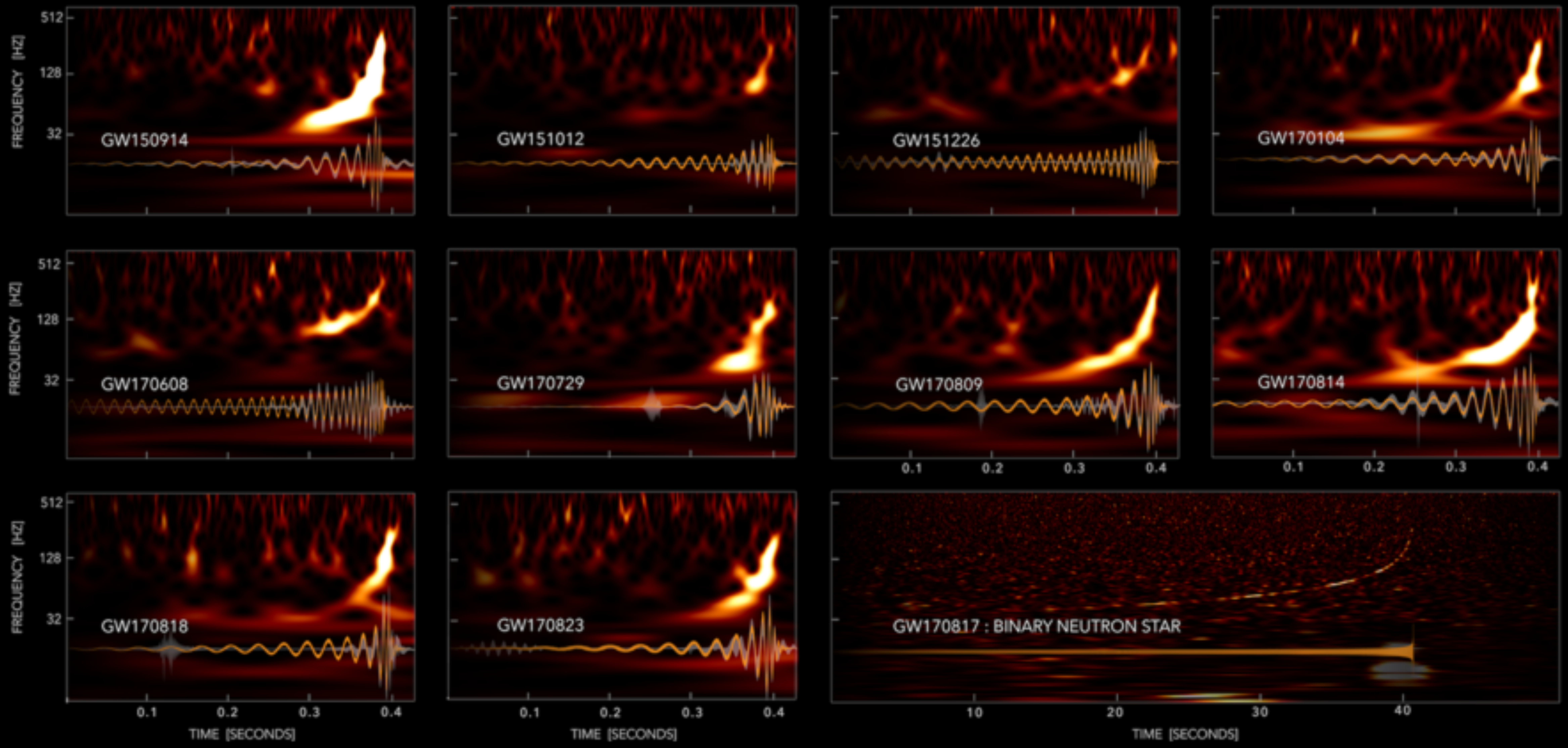






GRAVITATIONAL-WAVE TRANSIENT CATALOG-1

LIGO VIRGO Georgia Tech



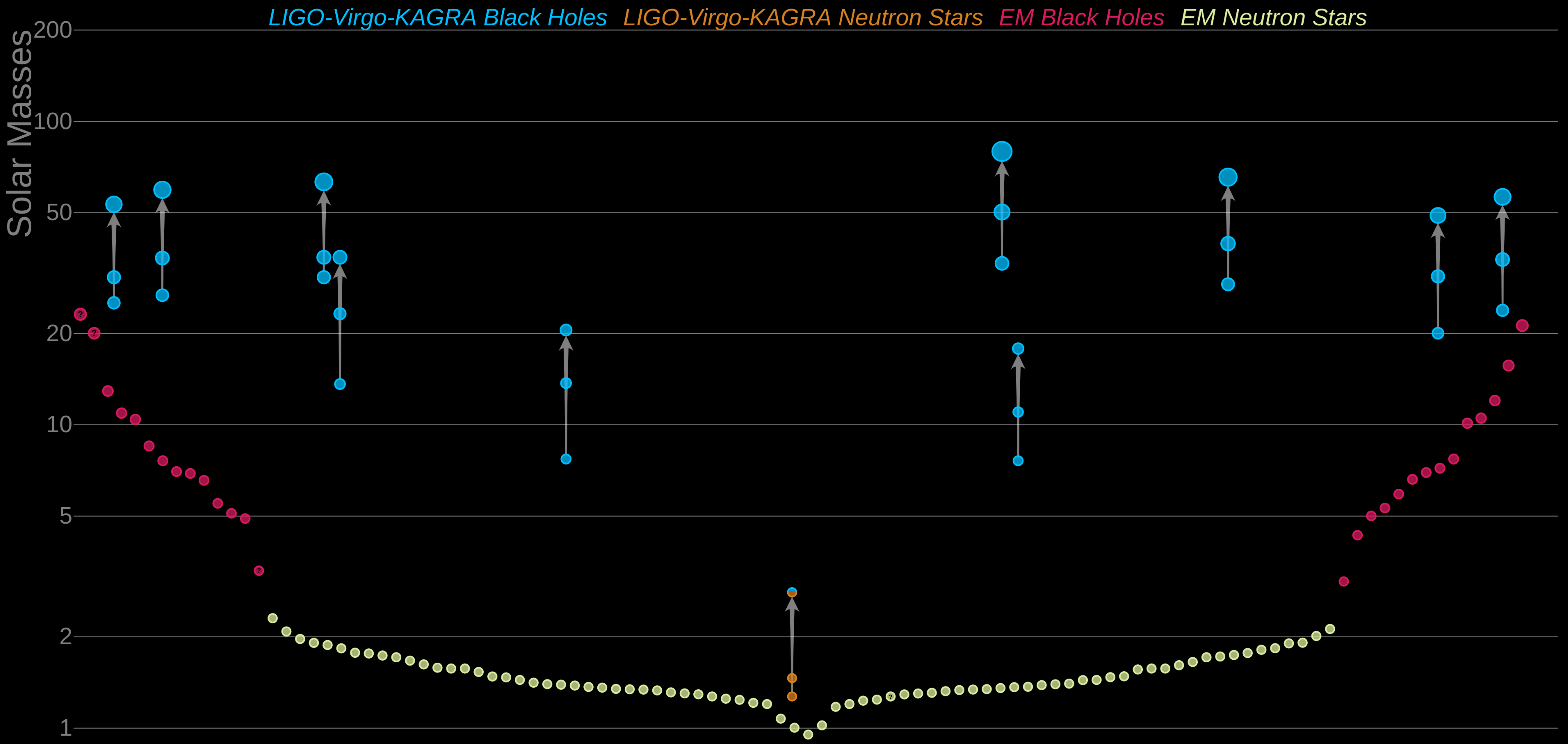
LIGO-VIRGO DATA: HTTPS://DOI.ORG/10.7935/82H3-HH23

WAVELET (UNMODELED) EINSTEIN'S THEORY

S. GHONGE, K. JANI | GEORGIA TECH

Masses in the Stellar Graveyard

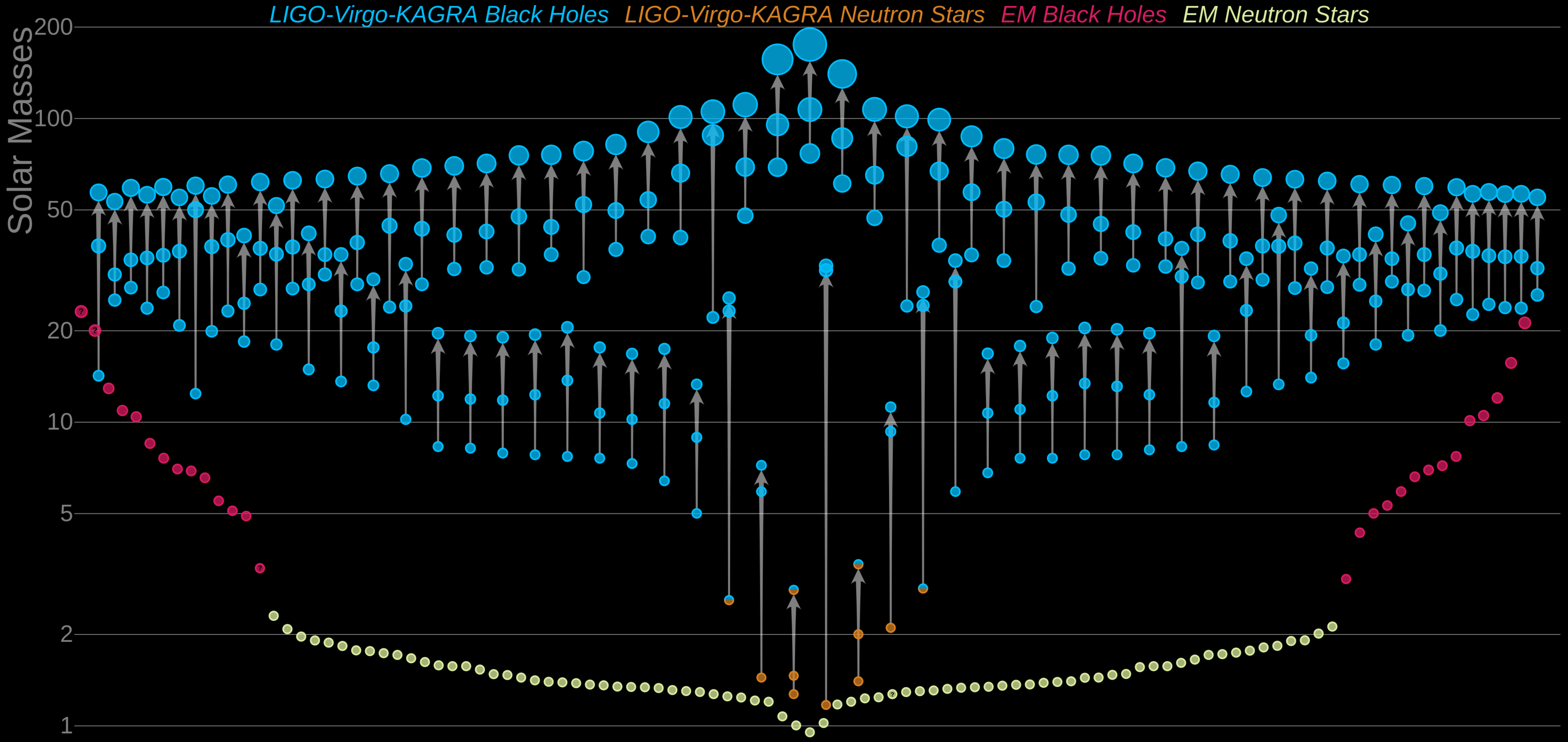
GWTC-1



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

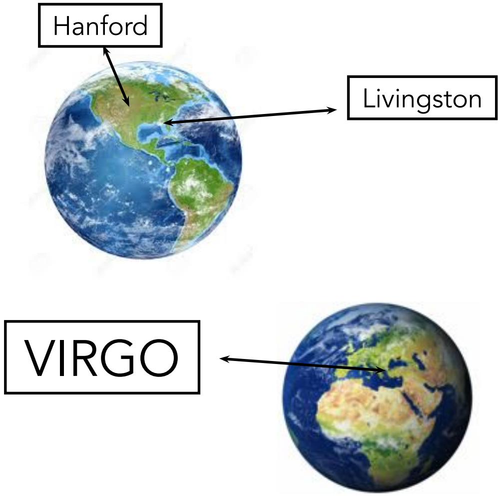
Masses in the Stellar Graveyard

GWTC-3

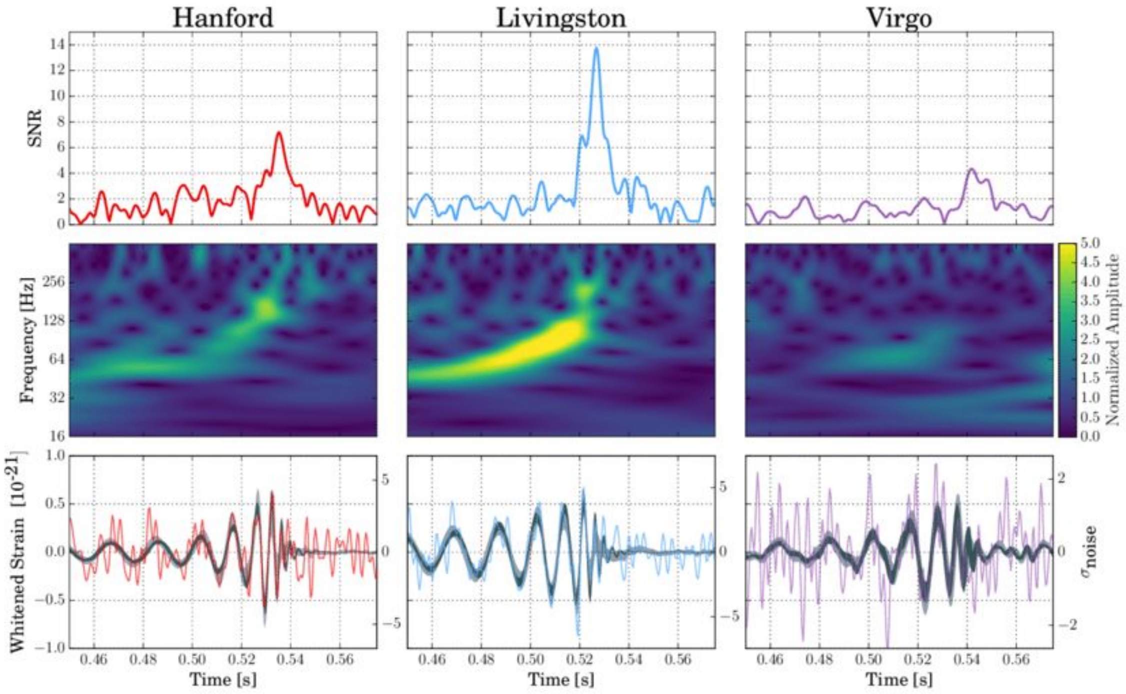


LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

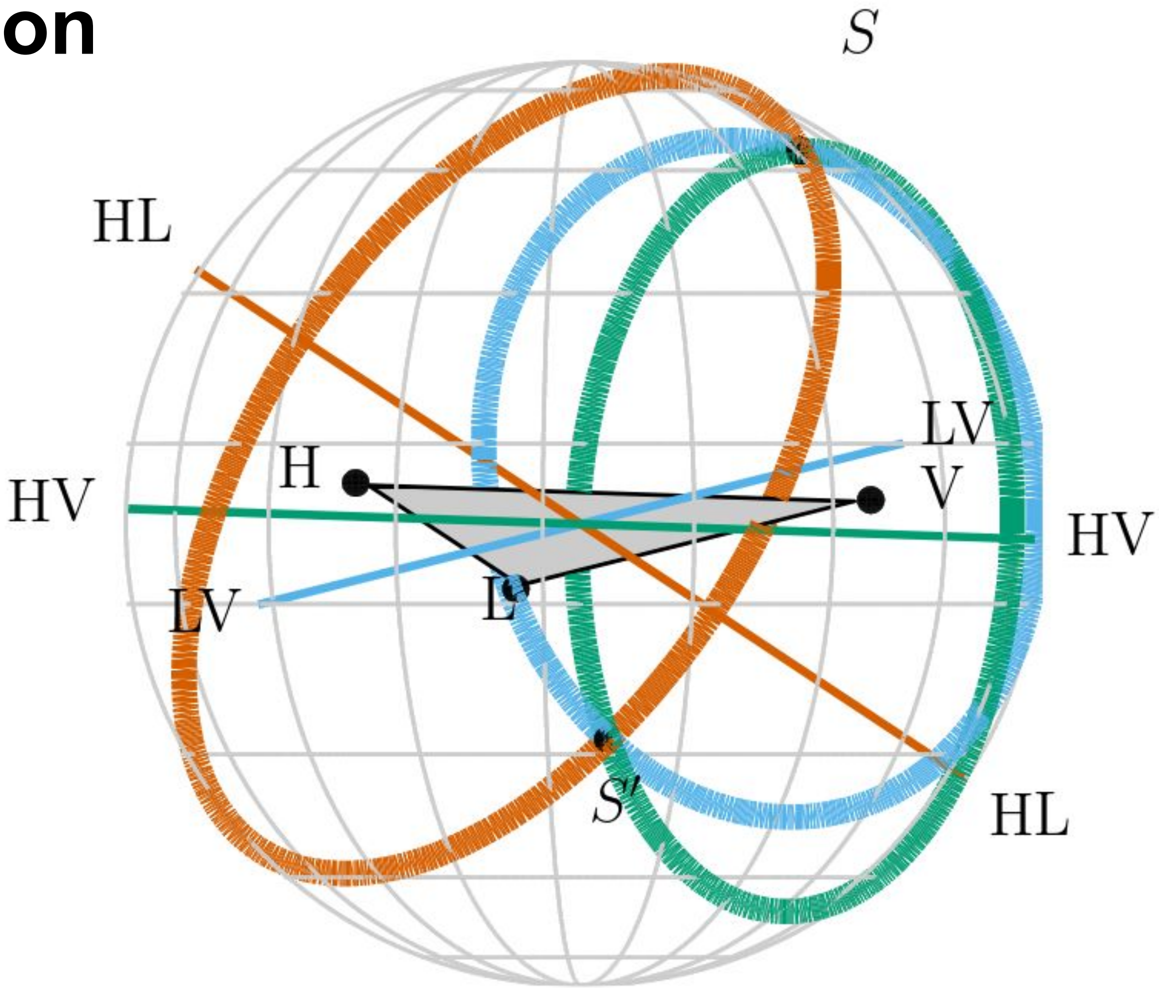
GW170814: A three detectors observation

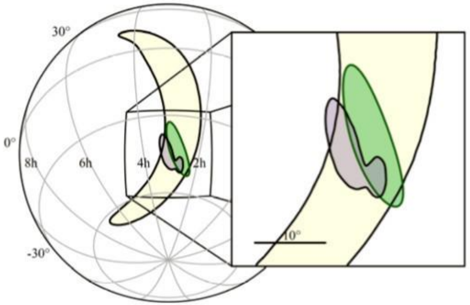


8 ms	0 ms	14 ms
SNR=7.3	SNR=13.7	SNR=4.4



Localization





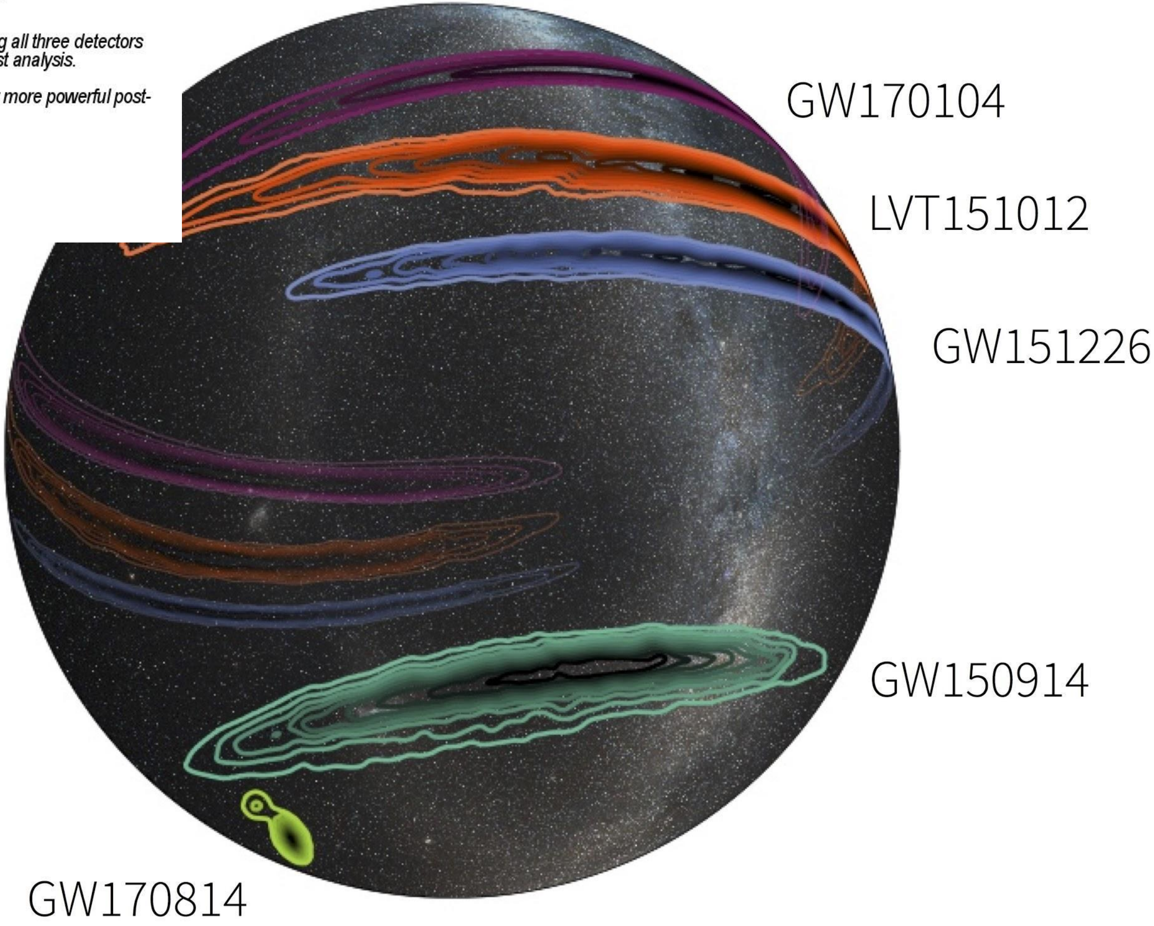
Gravitational wave source localisation.

Yellow : Localisation obtained using two LIGO detectors only.

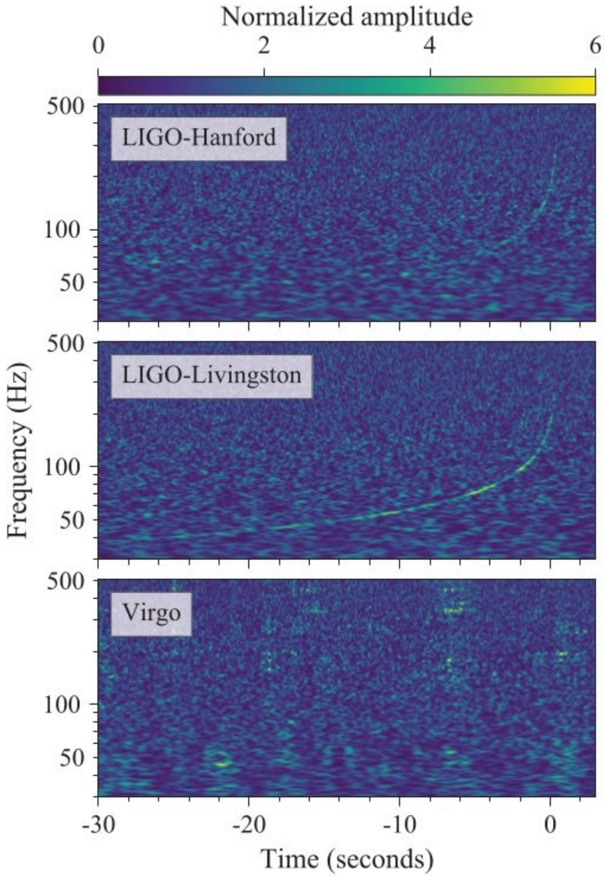
Green : Localisation obtained using all three detectors LIGO and Virgo, from real-time fast analysis.

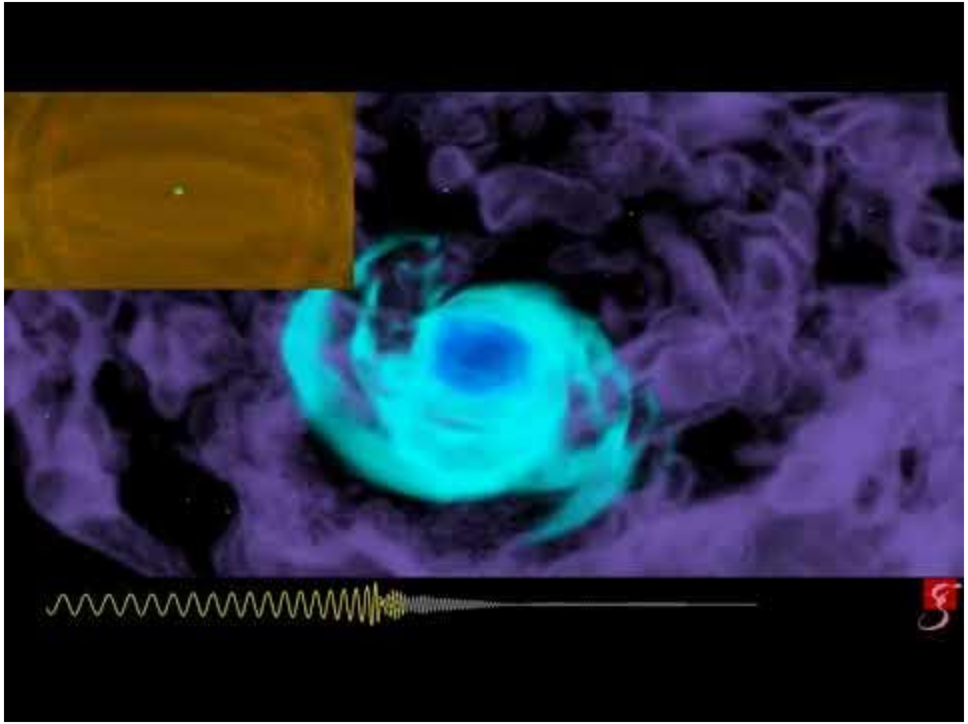
Mauve: Localisation obtained after more powerful post-analysis

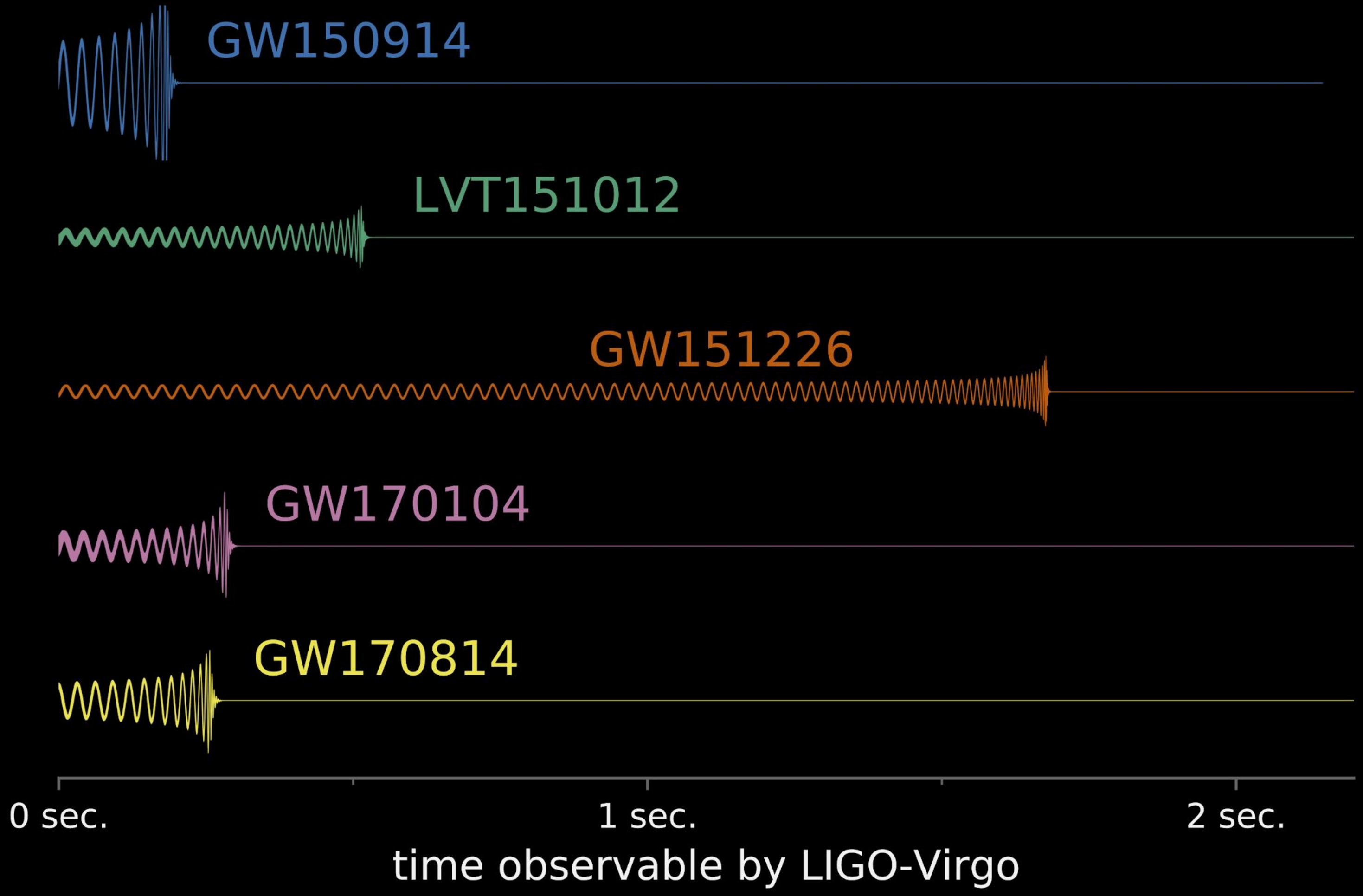
(c) collaboration LIGO-Virgo

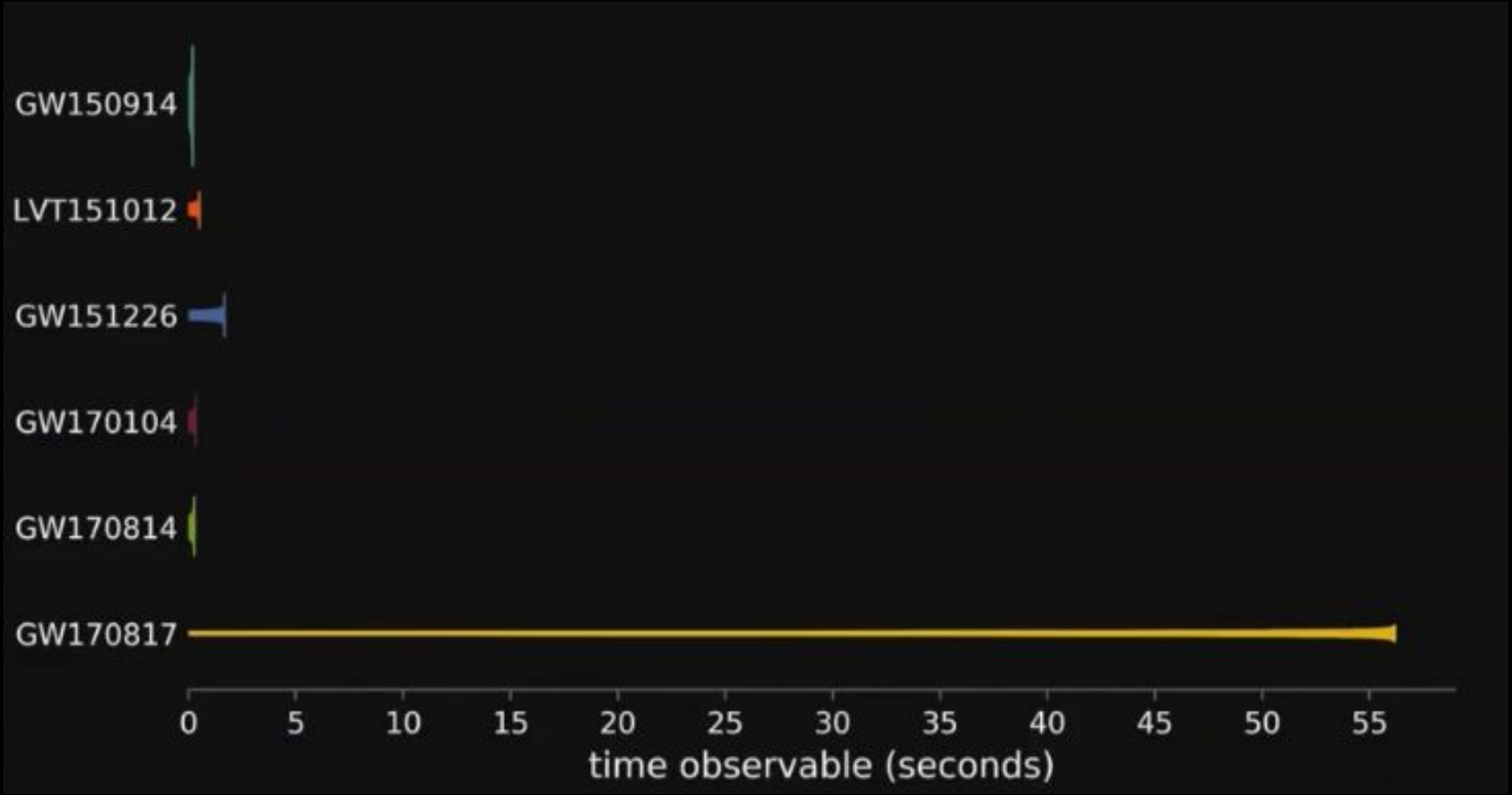


GW170817: binary neutron star inspiral

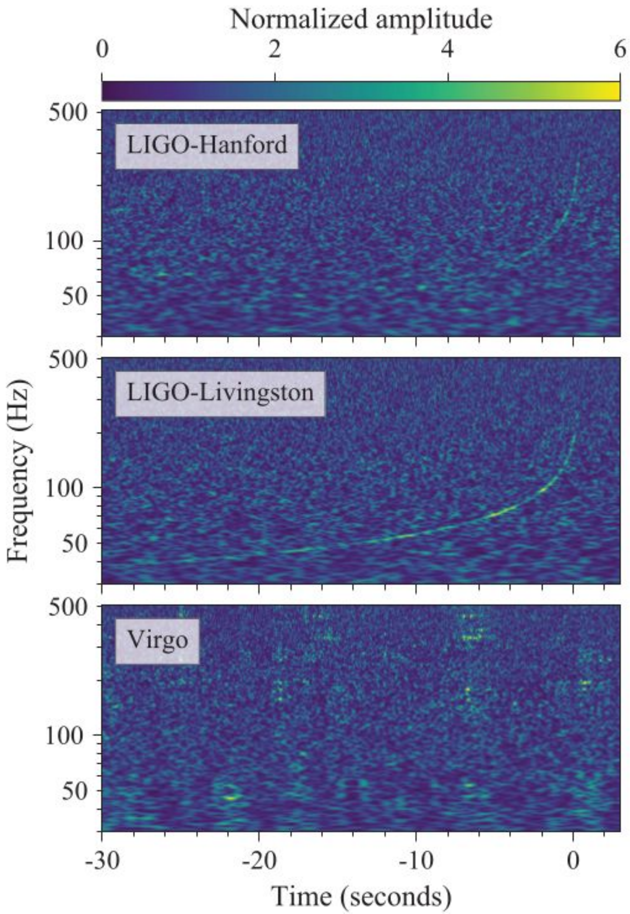




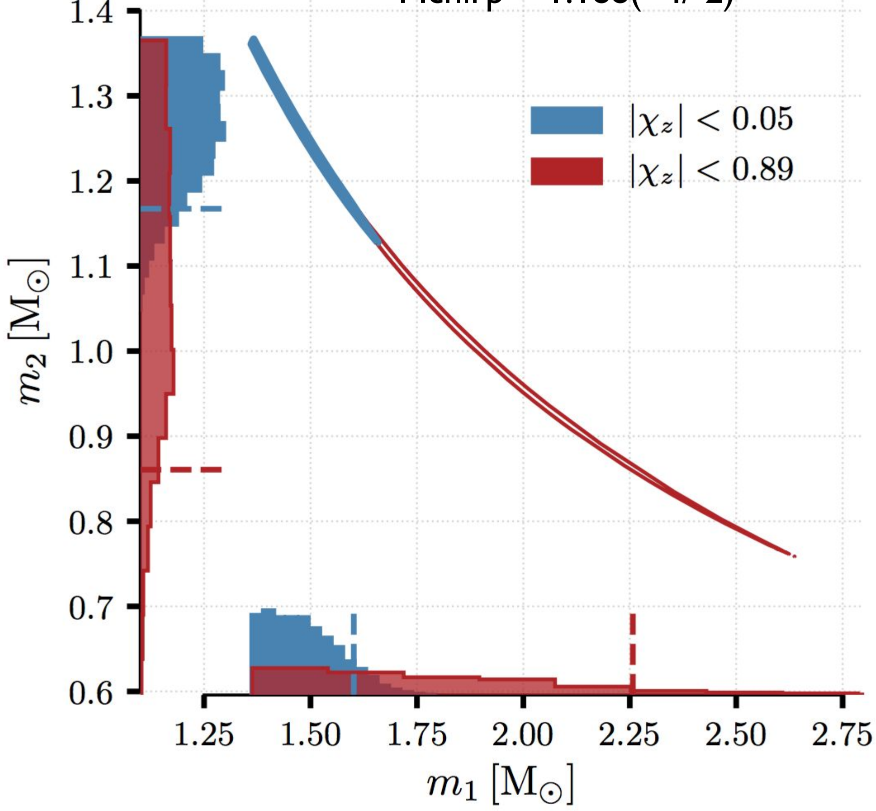

GW150914
LVT151012
GW151226
GW170104
GW170814
0 sec.
1 sec.
2 sec.
time observable by LIGO-Virgo


GW150914
LVT151012
GW151226
GW170104
GW170814
GW170817
0
5
10
15
20
25
30
35
40
45
50
55
time observable (seconds)

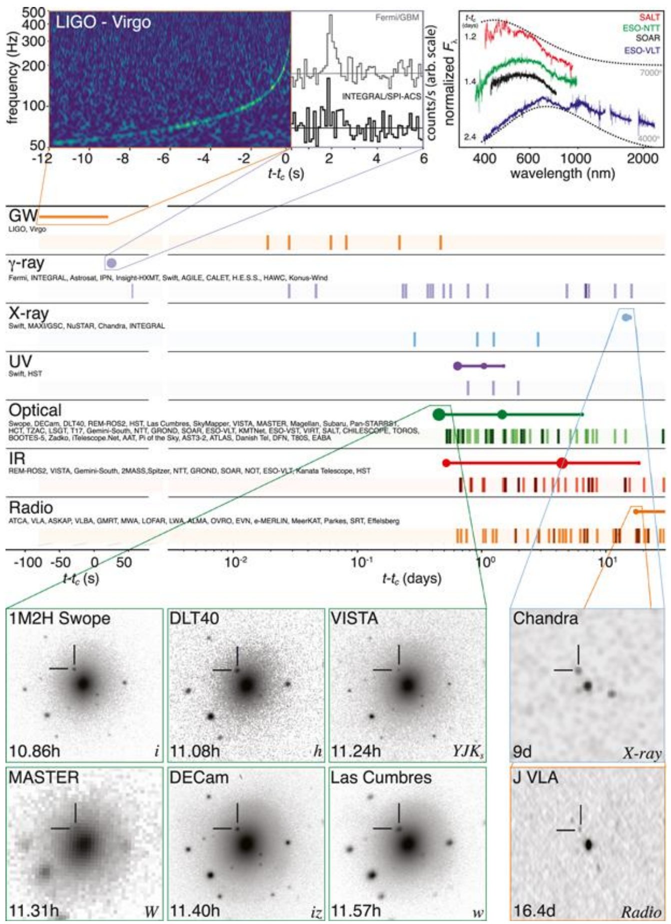
GW170817: binary neutron star inspiral

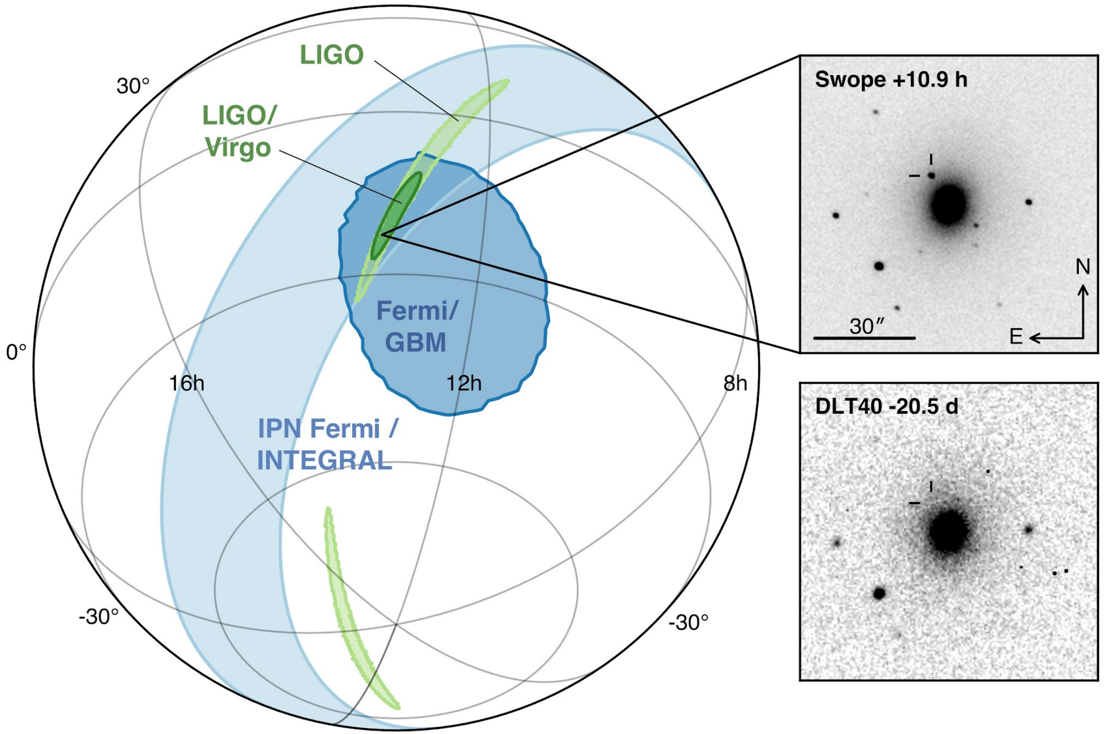


Mchirp = 1.188(+4/-2)

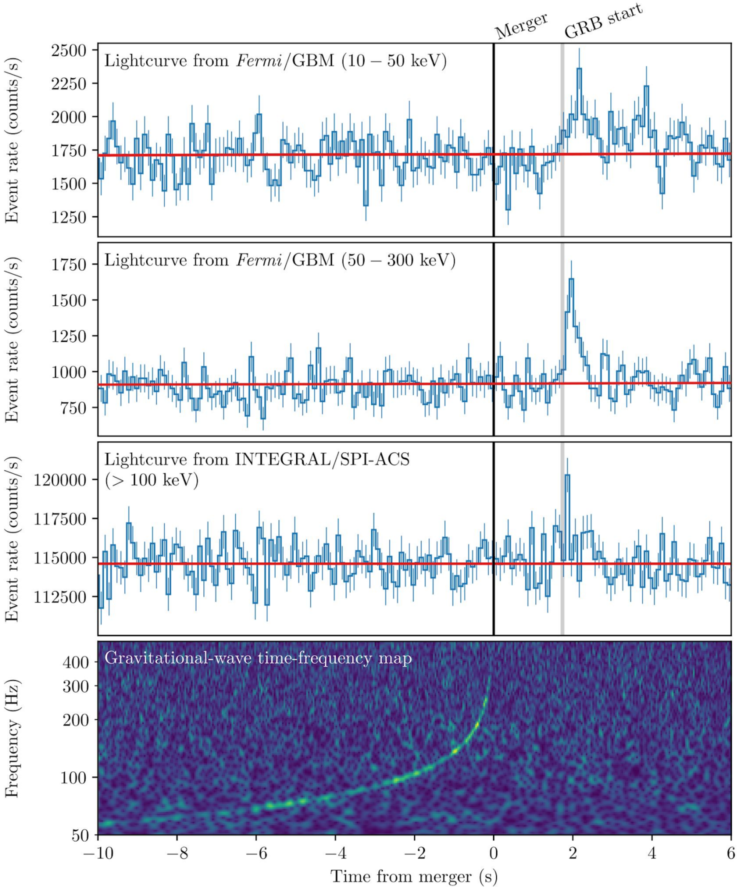


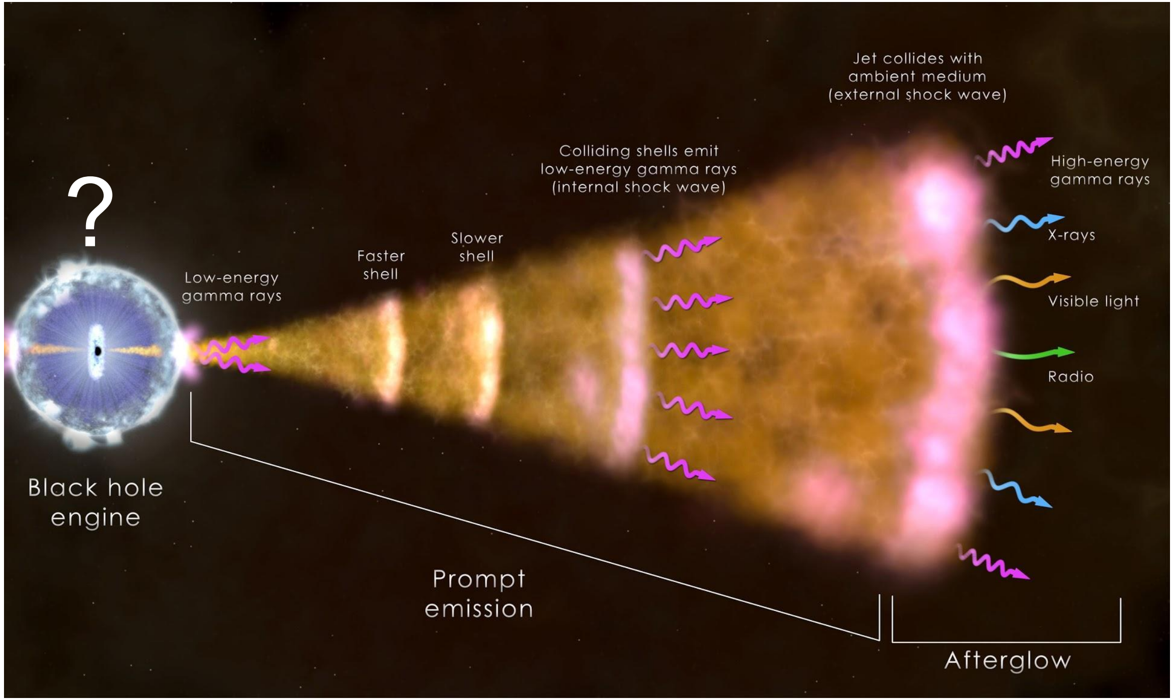
GW170817: multi-messenger observations





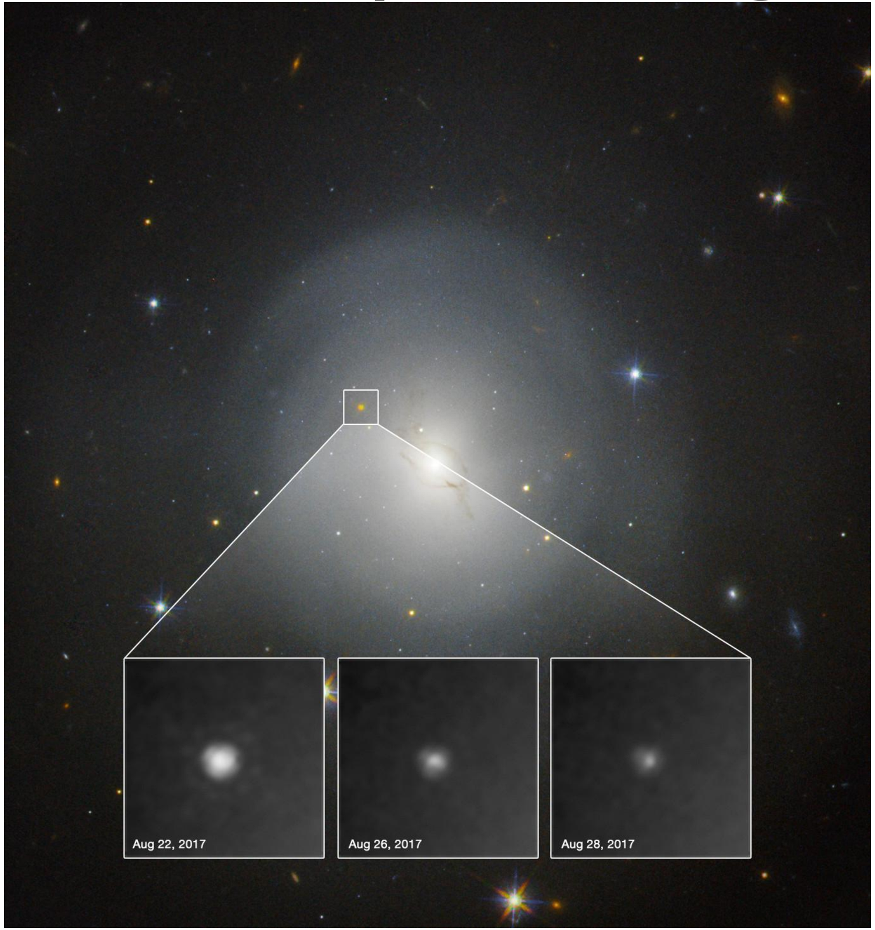
GRB170817: Gamma-ray burst

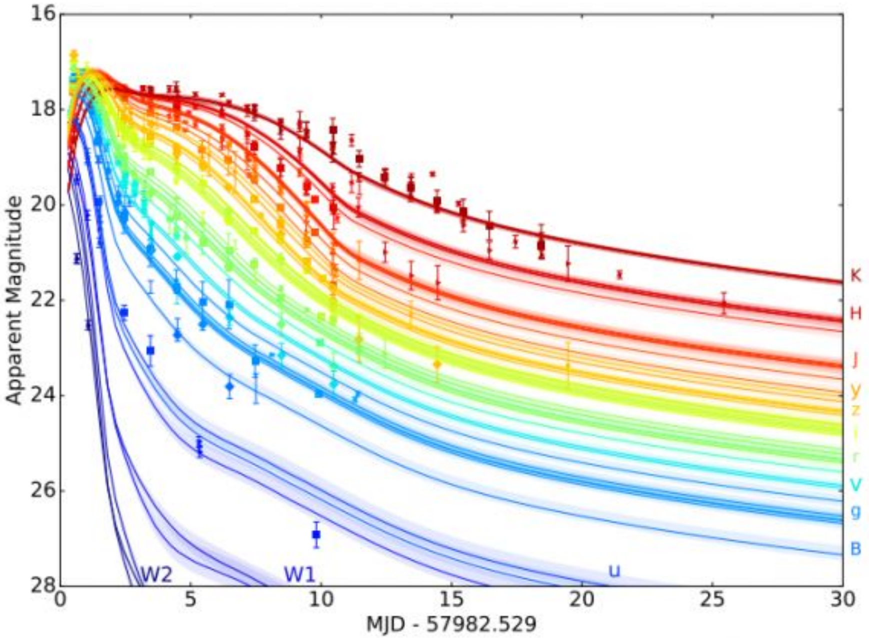




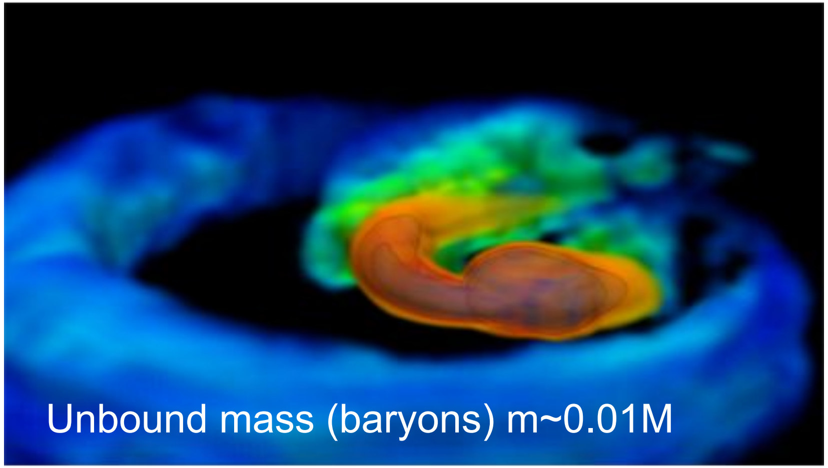
Evidence for the connection: GRB <==> Compact binary mergers

Kilonova (AT2017gfo)

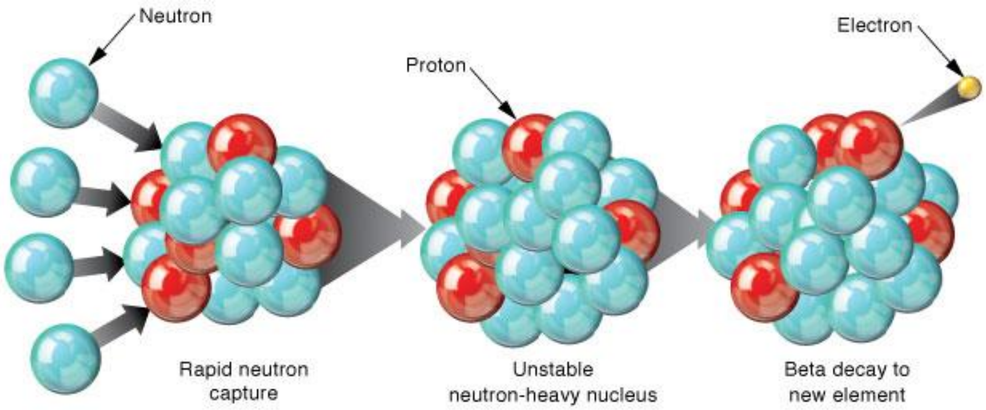




Mass ejecta, r-process and … shine !



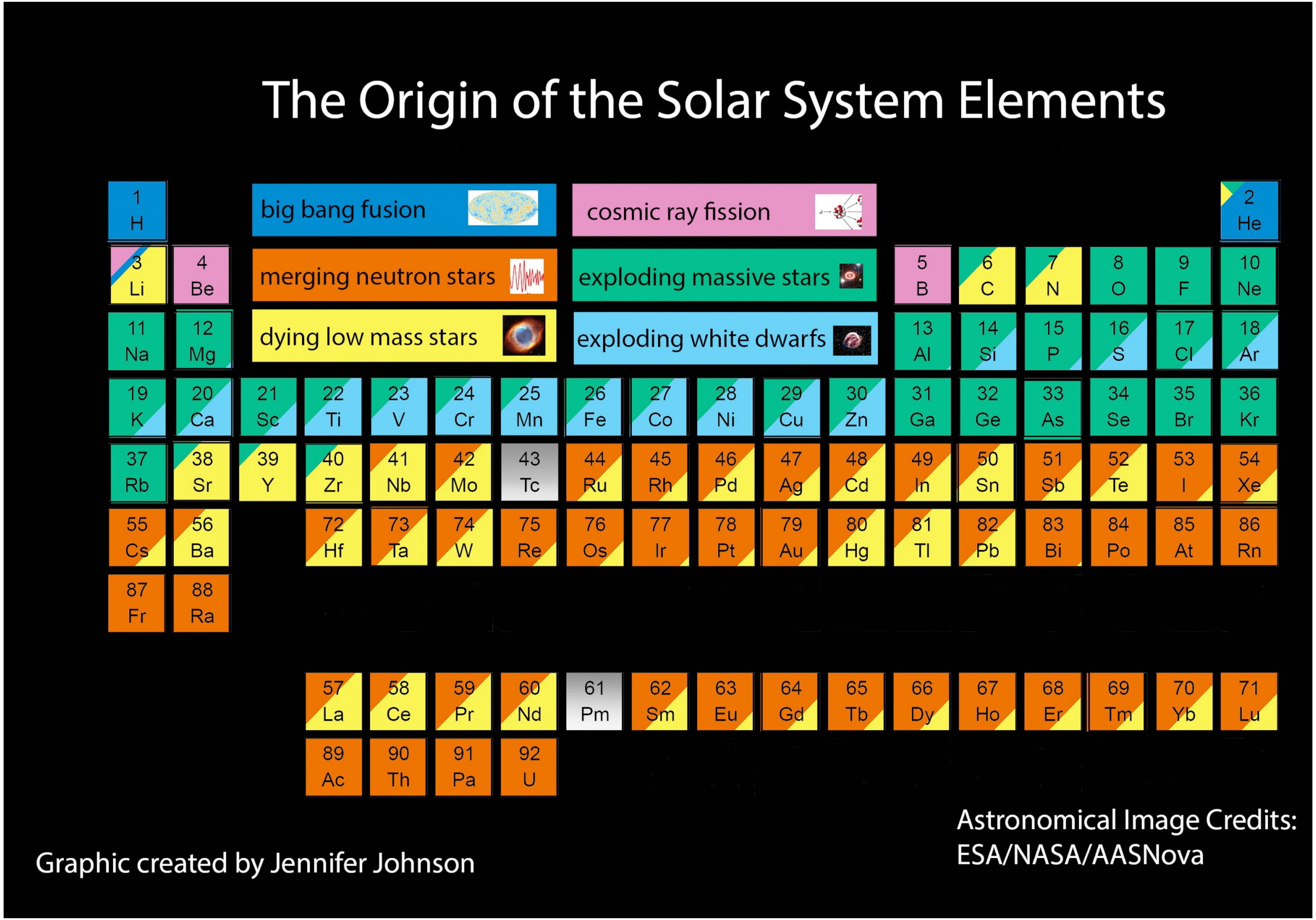
NS-BH and NS-NS collisions →
decompression of cold neutron star matter
(1974) D. Schramm, J. Lattimer …
… D. Eichler, T. Piran, F. Thielemann,
S.Rosswog and many others



e^- Thermalization & Black body radiation

- Radioactive heating in expanding plasma (β decay, α decay, fission)
- r-process element opacity (10^6 absorption lines)
- Detailed atomic and ionization abundances (composition)

→ challenges for nuclear and atomic physics



The Origin of the Solar System Elements
big bang fusion
cosmic ray fission
merging neutron stars
exploding massive stars
dying low mass stars
exploding white dwarfs
1 H
2 He
3 Li
4 Be
5 B
6 C
7 N
8 O
9 F
10 Ne
11 Na
12 Mg
13 Al
14 Si
15 P
16 S
17 Cl
18 Ar
19 K
20 Ca
21 Sc
22 Ti
23 V
24 Cr
25 Mn
26 Fe
27 Co
28 Ni
29 Cu
30 Zn
31 Ga
32 Ge
33 As
34 Se
35 Br
36 Kr
37 Rb
38 Sr
39 Y
40 Zr
41 Nb
42 Mo
43 Tc
44 Ru
45 Rh
46 Pd
47 Ag
48 Cd
49 In
50 Sn
51 Sb
52 Te
53 I
54 Xe
55 Cs
56 Ba
72 Hf
73 Ta
74 W
75 Re
76 Os
77 Ir
78 Pt
79 Au
80 Hg
81 Tl
82 Pb
83 Bi
84 Po
85 At
86 Rn
87 Fr
88 Ra
57 La
58 Ce
59 Pr
60 Nd
61 Pm
62 Sm
63 Eu
64 Gd
65 Tb
66 Dy
67 Ho
68 Er
69 Tm
70 Yb
71 Lu
89 Ac
90 Th
91 Pa
92 U
Graphic created by Jennifer Johnson
Astronomical Image Credits:
ESA/NASA/AASNova

Cosmography

Velocity-Distance Relation among Extra-Galactic Nebulae.

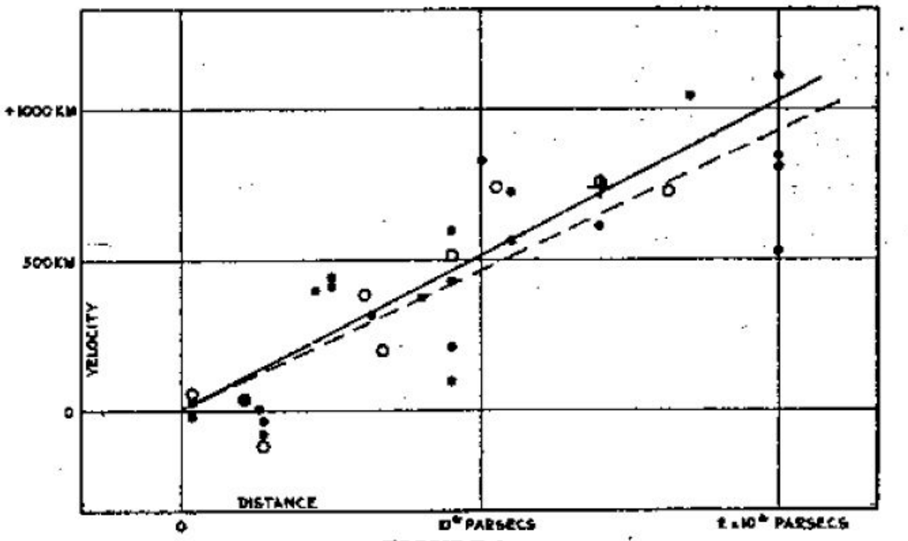
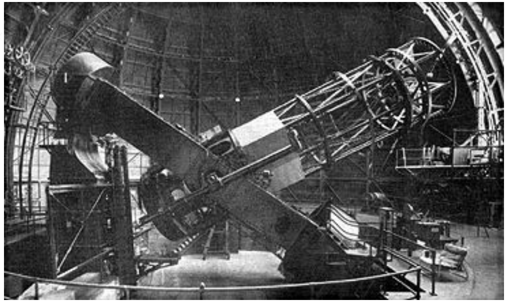


FIGURE 1



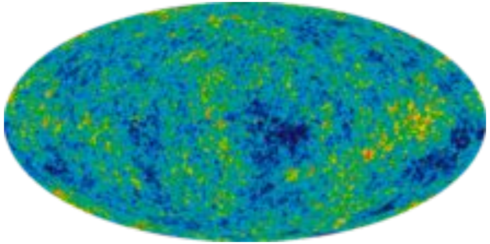
Type Ia Supernovae (ShoES)

H0 = 73.24 ± 1.74 km s−1 Mpc−1

Reiss+ arxiv:1604.01424

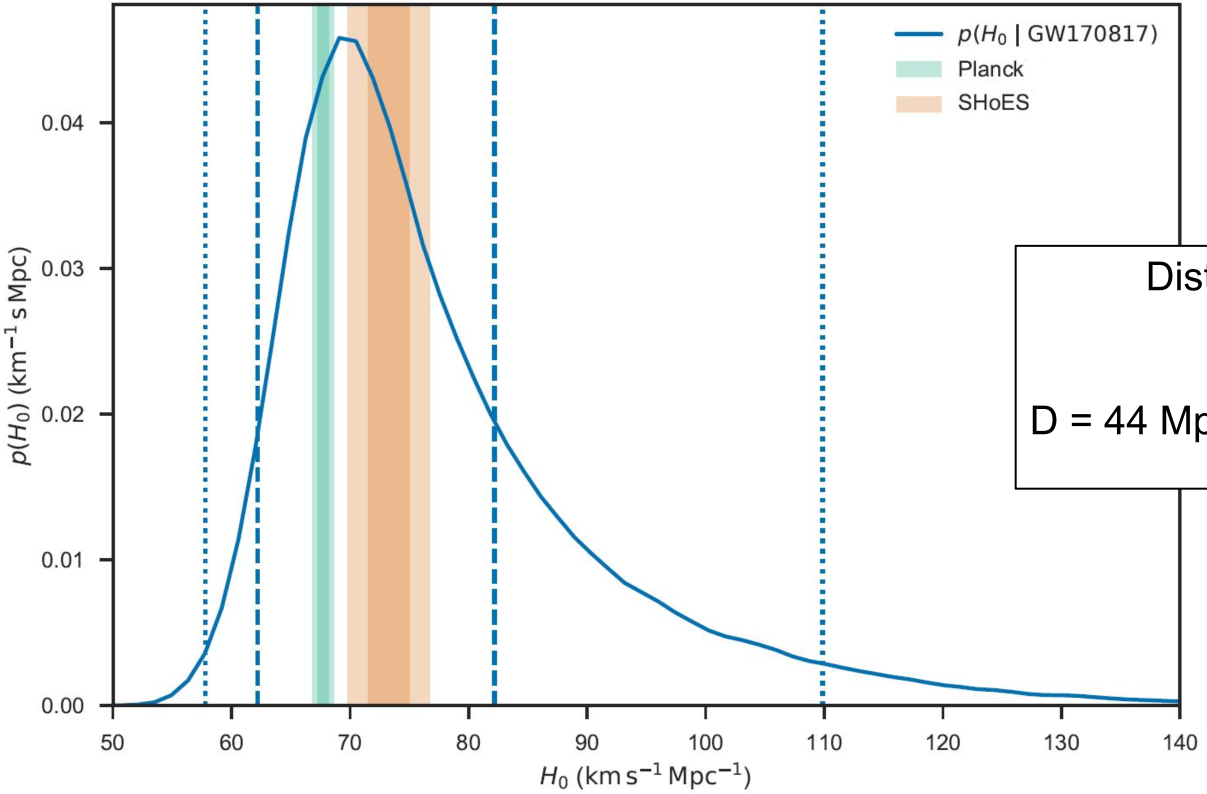
CMB (WMAP, Planck)

H0 = 69.3 ± 0.7 km s−1 Mpc−1



Standard sirens

[B.Schutz Nature 1986]



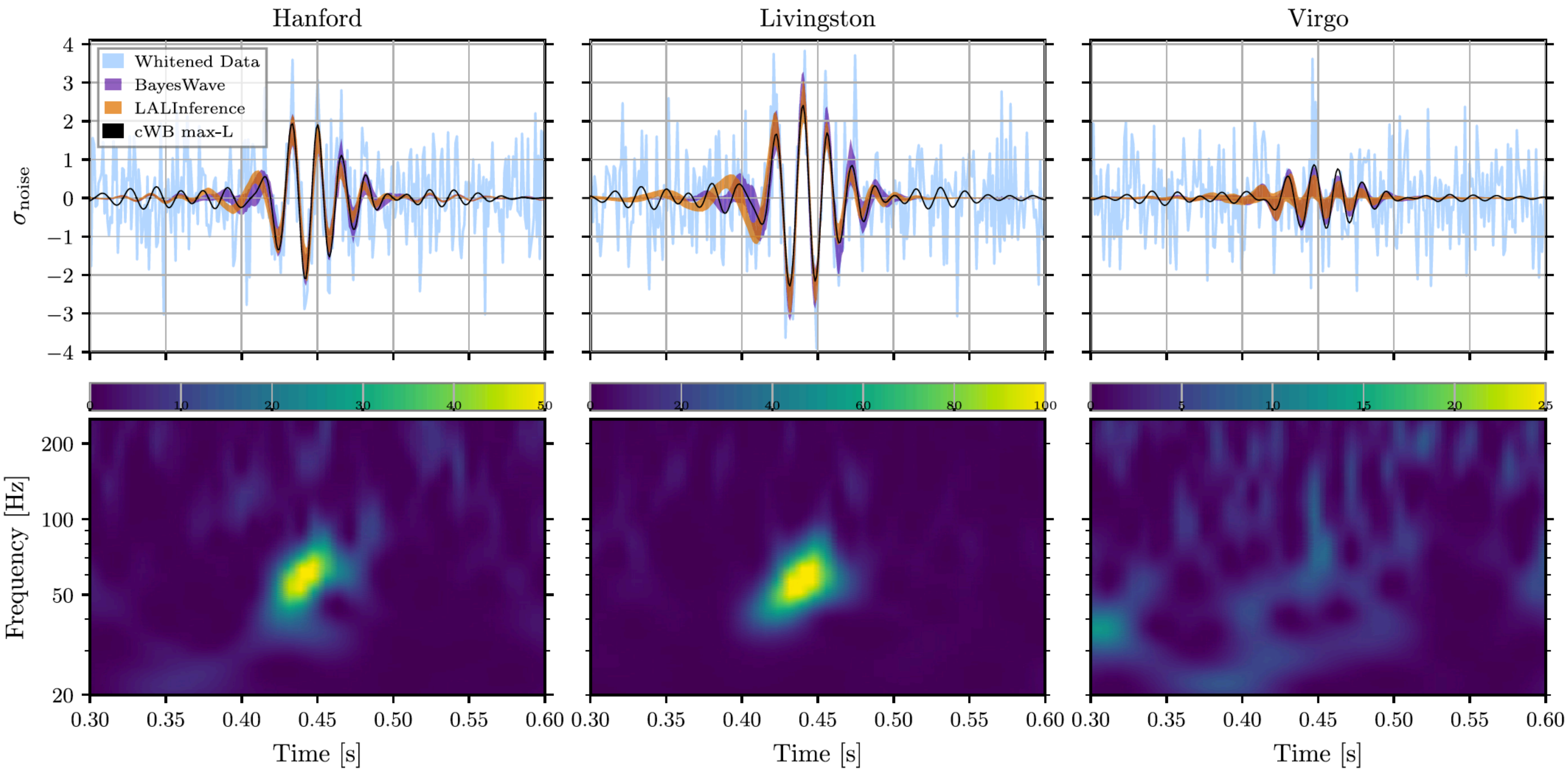
V = H0 D

Distance from the GW analysis

GW170817

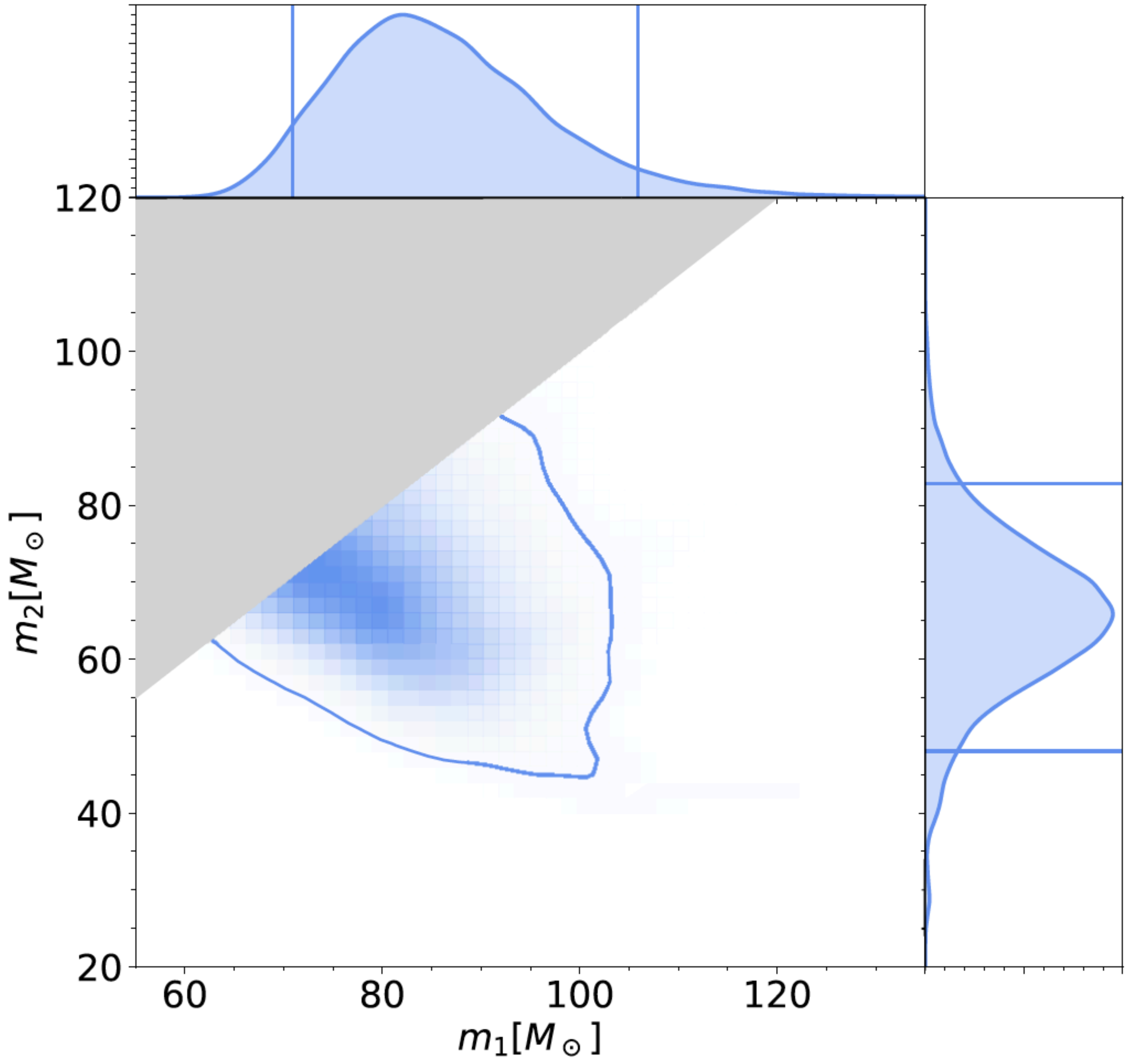
D = 44 Mpc (15%) Assume sky position

GW190521



Hanford
Livingston
Virgo
Whitened Data
BayesWave
LALInference
cWB max-L
$\sigma_{\rm noise}$
Frequency [Hz]
Time [s]

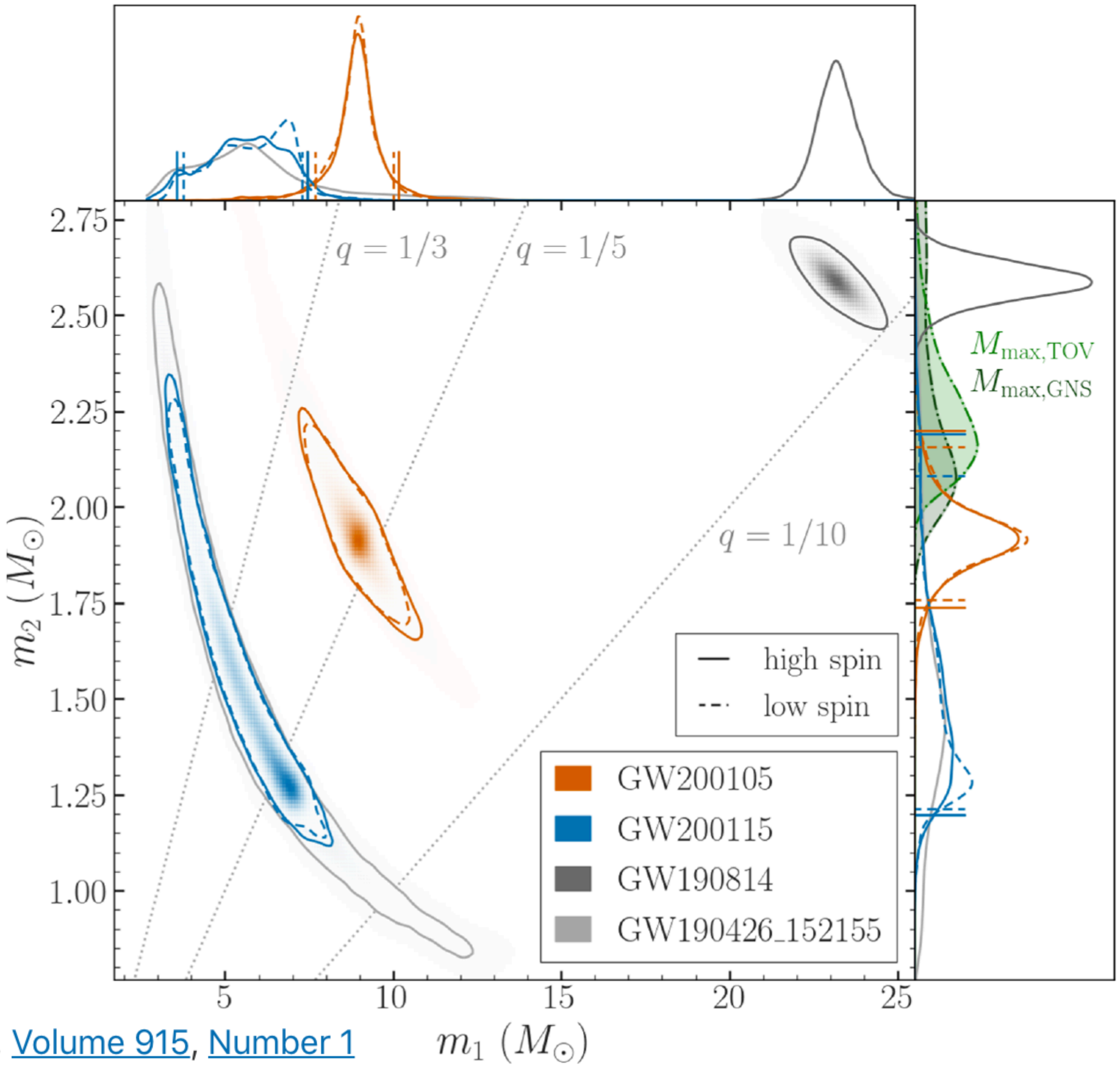
GW190521



- **Pair instability mass gap**
- **Links to galaxy formation**
- **Eccentricity**

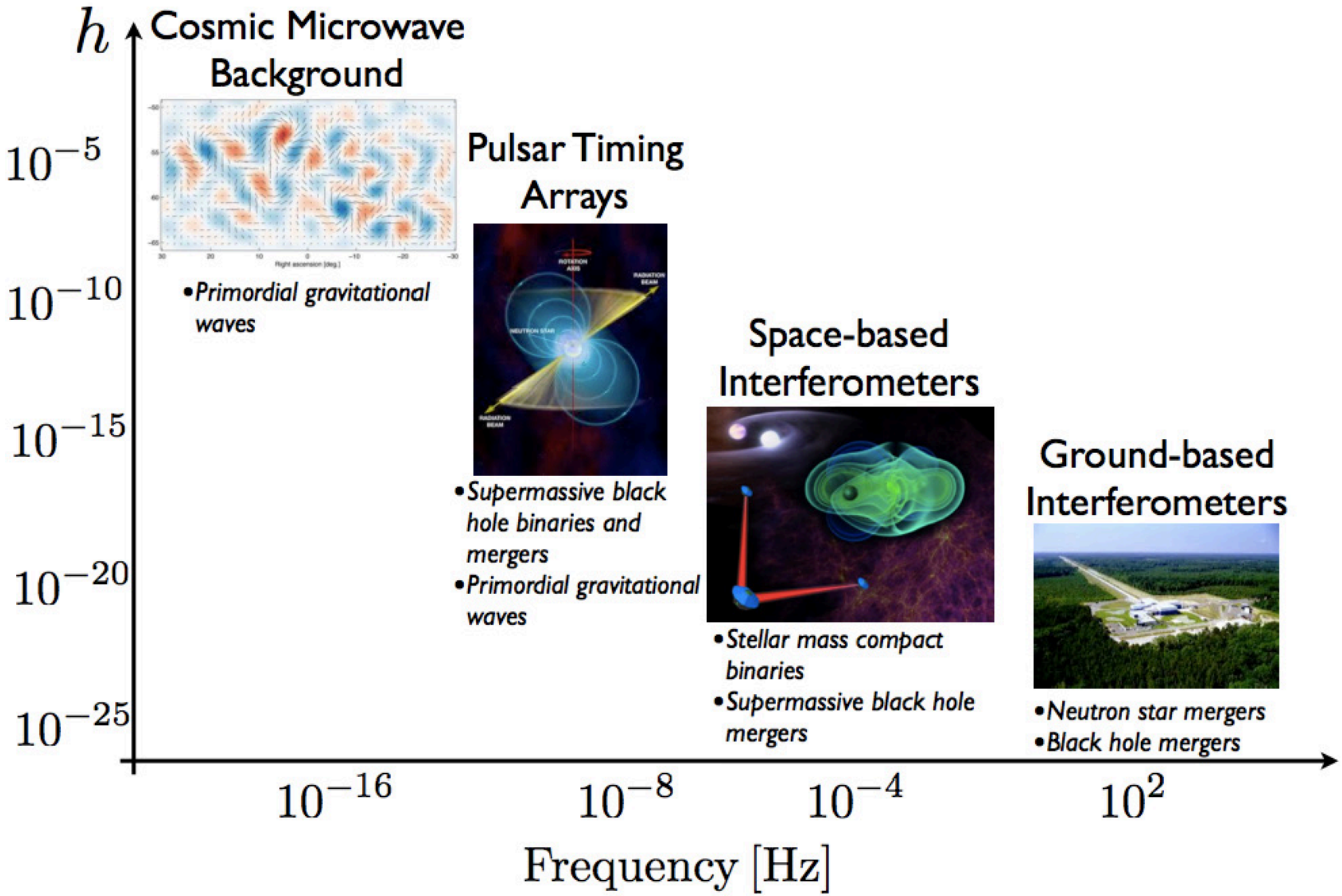
PHYSICAL REVIEW LETTERS **125,** 101102 (2020)

Neutron star - black hole binary



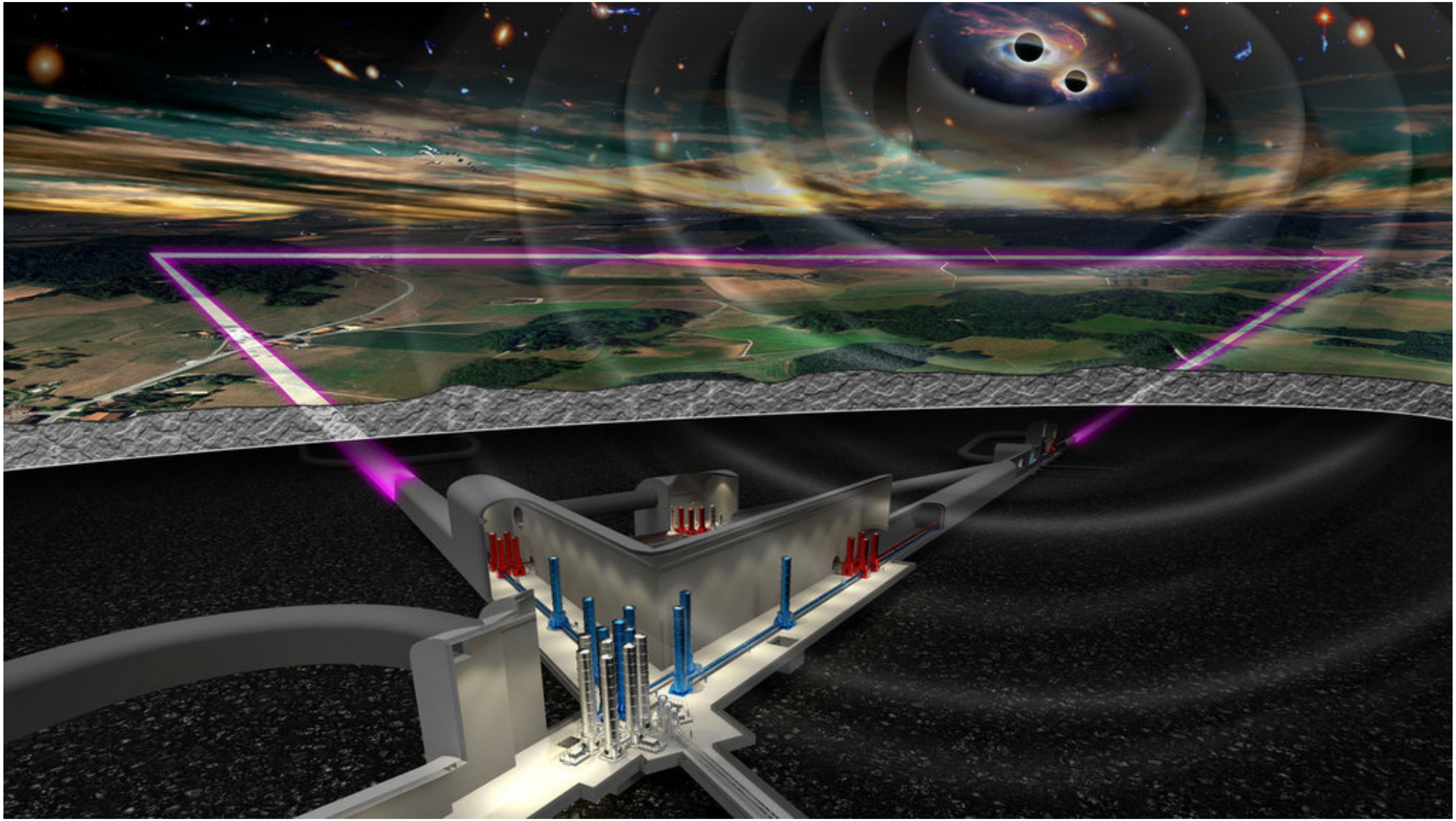
The Astrophysical Journal Letters, Volume 915, Number 1

GW spectrum

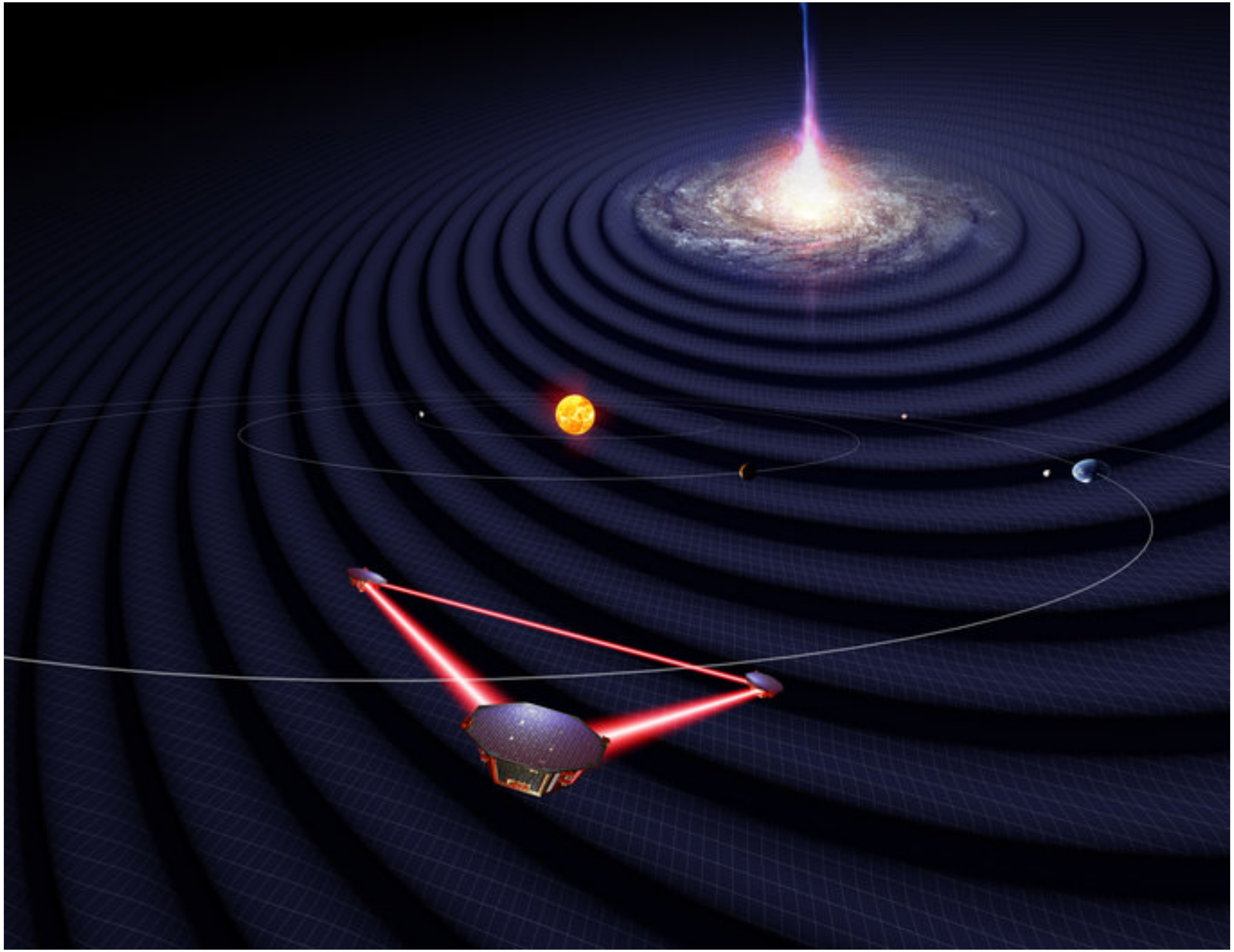


h
Cosmic Microwave Background
•Primordial gravitational waves
Pulsar Timing Arrays
•Supermassive black hole binaries and mergers
•Primordial gravitational waves
Space-based Interferometers
•Stellar mass compact binaries
•Supermassive black hole mergers
Ground-based Interferometers
•Neutron star mergers
•Black hole mergers
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-16}
10^{-8}
10^{-4}
10^{2}
Frequency [Hz]

Einstein Telescope



LISA



Fundamental physics

Is the graviton massive?

(Best bound available comes from GW)

Do GW travel at the speed of light? Do they disperse?

Are they only tensorial?

Are these objects actually black holes?

Can we use these signals to understand dark matter?

Is General Relativity a correct description of gravity,

in the dynamical and large curvature regime of these mergers?

Big bang / early universe

Take a walk on the dark side